Gödel's Incompleteness Theorem, the Liar Paradox, Refuting some Paul Davies' Statements and All That These Imply

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I. Introduction.

Paul Davies is the author of [I] "The Mind of God: The Scientific Basis for a Rational World." Davies is a "Templeton Prize" winner. This prize was, at one time, to be given for significant "Progress in Religion." However, the implications of Davies' work is a "giant leap backwards." Indeed, refuting various implications of his pronouncements indicates that gross errors are continually being made in the awarding of this prize.

This article concentrates on the true meaning of the Gödel results, how they are misunderstood and incorrectly interpreted, and the fact that they are not examples of the Liar paradox. A simple example is given that demonstrates the Paris-Harrington result and this result is used to refute Davies' Turing machine hypothesis. It is shown that the ultimate "answer" is comprehensible by rational means. It is explicitly shown that aspects of the ultimate cause are not mysterious, in the sense they are impossible to explain, but rather there is an explicit reason why, at present, they are incomprehensible.

II. Formal Verses Informal and Gödel Type Results.

(This section, especially the latter part, will require some mental effort. I know of no way to further simplify these portions and remain correct. So, they should be considered slowly and carefully.) In [5], I discuss certain facts about how mathematicians arrive at their conclusions. There appears to be no fully describable set of rules or processes that I "informally" apply to know in advance that a particular *mathematical statement* can be established successfully. That is, obtain an acceptable informal theorem. I simply *know intuitively,* in most cases, when a particular statement is probably provable as *mathematical "fact,"* and whether I can successfully establish the result. This I have done thousands of times. Davies discusses the Gödel incompleteness results and as usual Davies and his reviewers extend this to areas where it should not be applied. The Gödel results are about *formal mathematics* and formalizable expressions. However, there are no such incompleteness results unless *informal mathematics* is applied. (In order to aid memory, when a definition or description for a significant term is introduced the term will be written in **bold** typeface.)

After **formal mathematics** is described, it should be self-evident that formal mathematics does not correspond to the mathematics to which most of the world is exposed. Formal mathematics deals with finite strings of symbols termed **symbol-strings** written left-to-right in a very special manner. It takes a certain level of intelligence and informal descriptions to write a formal symbol-string. The same intelligence can write a computer-program that generates a correct but finite list of acceptable symbol-strings [WS-4]. (WS-n refers to a list, at the end of this article, of website references.) Although, like other areas in mathematics, the symbol-strings used are abstracted from real collections of meaningful phrases and sentences, after they are written, no meaning, no content is given to any of them. They are simply *forms* written left-to-right. And, they require that this intuitive notion of *left-to right* be comprehended. (They can be considered as written from right-to-left as long they are consistently expressed in this intuitive direction.) Later, a more detailed description for informal mathematics is given. Rather simple informal rules allow one to write down symbol-strings on a piece of paper or a computer screen so that they correspond to those expressed by others. Formal mathematics cannot be done unless one can follow informal rules. The symbol-strings are called **formula** (or *well-formed formula, wff* or *wf*) when they are generated by a set of specific rules. Sometimes, the formulas used are selected from a specific set constructed from a more general set of rules. The formal language $\mathcal{L}$, composed of all of the specifically formed symbol-strings, along with certain informally stated processes that yield "deductions" comprises a **first-order predicate calculus**.

In certain forms, the symbol-strings can be interpreted as meaningful sentences from an ordinary language such as English. Indeed, in **Nonstandard Analysis** this is a basic process, translating from informal mathematics to formal symbolism and conversely. I've done this hundreds and hundreds of times (WS-1). In general, only specific rules yield symbol-strings in a required order. This is accomplished by human thought and experience, or by machines programmed to properly assemble symbol-strings. However, the original production of Gödel type results requires that the rules be scrutinized. To construct a **formal proof**, formulas are written with step numbers that indicate where the formula is inserted into the list and a reason is stated that allows the formula to be placed at a particular step in the list. This yields a *finite* list of formula, where the formula at the last step is called a **consequence** (deduction, etc.) from a nonempty set of hypotheses. You also have deductions from an empty set of hypotheses. But, in this case, the last step is often termed a **theorem**. The last step in the finite list is said to be **deduced** from a set of hypotheses, where members from a set of hypotheses, if they exist, can be inserted as steps. The entire finite list is called a **proof**.

The entire collection of formula obtainable in this manner is termed a **formal theory**. Below is an actual list using symbols and rules for a *formal theory of natural numbers.* It uses rules of inference, various formal axioms and the formal symbolism of a first-order predicate calculus. A first-order predicate calculus also yields the rules used for classical-logic. These classical rules are used by various science-communities including the Physics-community. *Indeed, science-communities that interpret mathematical results physically should be able to state their propositions and deductions in an equivalent informal form.* However, this is not always the case. For example, the complete Feynman integral is not representable by an actual mathematical integral.

One list of **formal axioms** is $\mathbf{S} = \{S1, ..., S9\}$ [10, p. 117]. These axioms are supposed to generate logically the most useful part of the more than 3,000-year old informal theory of natural numbers. The fact is that **S** could never generate the entire informal theory since axiom S9 needs to be expression in $\mathcal{L}$ and Peano's informal **induction** axiom cannot be completely expressed by $\mathcal{L}$. (Note that $\mathcal{L}$ need only contain the specific symbols need to express **S**.) It uses the phrase "Any relation that has a certain property. . . ." The objects that comprise the class of all such relations is not specified. When informal induction is used, mathematics-communities determine whether such a relation is reasonable. On the other hand, informal statements obtained from the informal theory of natural numbers are translated using $\mathcal{L}$. One such collection is **S** and does not include the form "$(\forall x_1((x_1 = x_1)))$." For the natural numbers, this symbolic form is interpreted as stating that each natural number is "equal" to itself, a statement needed to obtain the usual notion of equality. This notion of **equality** means that equal natural numbers, represented say be a_1, a_2, can be substituted one-for-the-another within any formula and a completely equivalent formula results.

Using **S**, the following five-step proof yields the symbol-string $(\forall x_1((x_1 = x_1)))$ at step 5. (One need not "know" the exact meanings for the rules indicated in the right-side "reasons-column." This is just a demonstration of the *form* in which formal mathematics is presented. It obviously has a lot in common with a Euclidean geometry proof.) The symbol "x_1" below, represents a *variable* and when the allowed procedures are applied, the name of the axiom (i.e. a member of a set of hypotheses), rule or procedure applied is stated in the second column. The rules used are abbreviated in this proof. The rules MP and Gen are described informally. Unless you comprehend how to apply Gen, step 5 could not be achieved for **S**.

1. $(x_1 + 0 = x_1)$ . . . . Axiom S5.
2. $((x_1 + 0 = x_1) \to ((x_1 + 0 = x_1) \to (x_1 = x_1)))$ . . . . Axiom S1.
3. $((x_1 + 0 = x_1) \to (x_1 = x_1))$ . . . . MP(1,2).
4. $(x_1 = x_1)$ . . . . MP(1,3).
5. $(\forall x_1((x_1 = x_1)))$ . . . . Gen(4).

The above symbol x_1, and others that contain integer subscripts, are not the same as the symbols used to represent the natural numbers in $\mathcal{L}$. The above subscript is an abbreviation for $x_|$ (the Kleene notation). The x_2 is the same symbol as $x_{||}$, etc. The $\mathcal{L}$ formal symbols for the natural numbers are specifically defined using the **S** operator $'$. The formal symbols for the natural numbers are $\{0, 0', 0'', 0''', \ldots\}$. Other symbols like r are used to represent, via italic fonts, these formal symbols. When necessary roman fonts are used to represent arbitrary informal natural numbers like r and, in $\mathcal{L}$, this r is represented by r. As illustrated, this is a specific formal proof using a set of axioms, the allowed rules MP (modus ponens) and Gen (generalization) that yield the symbol-string $(\forall x_1((x_1 = x_1)))$. There are other proofs that yield the same consequence. Indeed, you can repeat step 1 a hundred times if you wish. But, this list cannot be properly constructed unless a rule's intuitive meaning is comprehended. Of course, these rules can be translated into a computer-language. The important word here is "translated." The methods require in-depth comprehension in order to translate them into a computer-language. This I did for many aspects of formal logic.

One might think it very strange to establish, by strict formal means, that $x_1 = x_1$, but the fact is that this formula is not one of the **S** axioms. Notice that the set of all theorems is part of a formal theory. There are various formal axiom-systems similar to **S** that yield the exact same formal theory and only that theory. Applying a first-order predicate calculus to a set of formal Peano Axioms produces Peano Arithmetic **PA**. The axiom-system **S** is such an axiom-system. Another axiom-system that yields **PA** is at [WS-2]. Such axiom-systems are said to be equivalent. Since the symbols used are content free, then a proof is a form of informal rule-controlled symbol-manipulation as used in abstract mathematics and, at this stage, nothing more than that. Notice that the notion of "true" or "false" is not used to obtain this formal proof.

Informal mathematics is mathematics expressed in an ordinary language such as the one I'm using to write this article. The words used have "meanings." The logical processes used are, usually, not mentioned. But, if challenged, they can be. The rules of inference are almost always a small collection of the common rules for deduction taken from the set of classical-logic rules. Originally, David Hilbert [7] illustrated the somewhat vague definition for the informal methods one should use. These are called **finitary methods.** After about 3,000 years of evidence, the Hilbert finitary methods that yield portions of the informal theory of natural numbers $\mathcal{N}$ have not produced a contradiction. So, those portions of $\mathcal{N}$ are assumed to be empirically consistent. Indeed, Hilbert uses this consistency assumption to prove the consistency and independence of various axioms for formal logic. He calls this the *the method of arithmetical interpretation.* But, depending upon the dictates of a mathematics-community, a mathematician is allowed to use other informal axiom-systems and maybe different procedures to study and obtain informal theorems that express properties of **PA**. Let $\mathcal{H}$ denote Hilbert's informal theory as used to informally establish results. Usually, $\mathcal{H}$ contains all the necessary portions of $\mathcal{N}$ needed to obtain results about **PA**. The informal language is called the **metalanguage**. The formal language is called the **object language**. In this case, $\mathcal{L}$. Usually, the metalanguage also contains the object language. These entities and informal procedures that yield conclusions about how proofs are produced or have other informally described properties is called **metamathematics.** The informally stated theorems are called **metatheorems**.

I mention again that *just to construct a formal proof, an individual must have a set of procedures described in an informal language.* Using a metalanguage to study a distinct formal language eliminates the accusation that metamathematical proofs are circular in structure. For example, consider any book that describes how one writes statements in a computer-programming language. The programming language is the object language and the instructions, along with the programming-language, is the metalanguage.

The famous Gödel results deal with formal proofs. These results are obtained via the acceptable Hilbert methods $\mathcal{H}$. (By-the-way, this is the same Hilbert who, at a meeting of mathematicians and physicists, presented a set of equations for a gravitation field theory that is equivalent to the equations Einstein presented 5-day later. The big difference is that Hilbert's equations are derived logically from a few axioms and Einstein's equations are obtained by an intelligent guess, a choice among many.) Any comprehensive book that establishes the Gödel results requires a vast amount of informal mathematics [10, pp. 129-162.]. Indeed, those who accept the Gödel results should agree that the informal rules for formal deduction can be properly describe by informally written formal-type statements. There is no incompleteness result for the formal theory of natural numbers unless one assumes that $\mathcal{H}$ and **PA** are *consistent.*

For $\mathcal{L}$, let $\neg$ informally denote a negation. Applying $\neg$ immediately to the left of any member of $\mathcal{L}$ still yields a member of $\mathcal{L}$. An axiom-system **D** is **formally inconsistent** if two proofs using **D** have as last steps the form X and the form $(\neg X)$, respectively, where X is any formula in $\mathcal{L}$. Putting these two proofs together into one proof and, for any pair of formulas $A,\ B$ in $\mathcal{L}$, adjoin the proof of the formal theorem $((\neg A) \to ((A \to B)))$. Two applications of MP, where $B = X$, yields A. Hence, if these two proofs exist, then there is a proof, where any formula from $\mathcal{L}$ is a last step. Moreover, in this case, there is a proof that yields $(X \wedge (\neg X))$ (The symbol $\wedge$ is informally translated as "and.") Technically, $(X \wedge (\neg X))$ is termed a **contradiction.** This formula also yields a proof for any member of $\mathcal{L}$. Many other formulas are equivalent to $(X \wedge (\neg X))$. An axiom-system that does not yield a contradiction is **(formally) consistent**. Since there are other ways to express a notion of consistency, this form is called **simple consistency**. The term consistent used throughout the remainder of this article means simple consistency.

Please note: *From the definition of consistency, requiring* **PA** *to be consistent implies that if X is a member of* **PA**, *then $(\neg X)$ is not in* **PA**, *that is, there is no proof for it using any* **PA** *axiom-system. Further, if $(\neg Y)$ is a formula in* **PA**, *then $(\neg(\neg Y))$ cannot be proved implies that Y cannot be proved since $(\neg(\neg Y))$ is equivalent to Y.* So, the Gödel theorem that states that a formula $\mathcal{G}$ cannot be the last step in any proof must be stating something more than what is already known. Indeed, in modern proofs, the Gödel result states that there is a formula $\mathcal{G}$, where $\mathcal{G}$ is equivalent to a formula in *explicit form*, and $\mathcal{G}$ cannot be formally proved using **PA**. But, there is still more. The Gödel approach also shows that under an additional requirement termed ω-consistency, the formula $(\neg G)$ cannot be a final step in a proof. If a formal axiom-system is ω-consistent, then it is formally consistent. After Gödel's result appeared, Rosser constructed a formula $\mathcal{R}$ only using the assumed consistency of **PA**, where neither the formula nor its negation can be formally proved using **PA**. The notion of **completeness** for an axiom-system means that *every* formula or its negation can be formally proved. So, the result states that **PA** is not complete or it is **undecidable**. This incompleteness persists even if various collections of specific axioms are adjoined to **S**. That is a much more significant fact.

What is Gödel's basic approach? To establish Gödel's Incompleteness Theorem, each member of $\mathcal{L}$ is *coded*, in a special way, using the Hilbert allowed properties of the informal natural numbers. The number that represents an object in $\mathcal{L}$ is called a **Gödel number**. Then all other procedures used to construct a formal proof are coded. Consider the reason stated for step 5 in the above proof, Gen. To get step 5, step 4 is used. So, this informal procedure is described by an expression that states that "the Gödel number y for the formula in step 5 comes from the Gödel number for $(x_1 = x_1)$ in step 4 by application of the rule Gen." Of course, this corresponds to some sort of mental procedure that allows us to follow the rule and get the form in step 5 from step 4. This statement is identified by the symbols $\mathrm{Gen(x,y)}$. It is written in terms of the metamathematical symbols. The following statement, in terms of Gödel numbers, is claimed to present the informally stated $\mathrm{Gen(y,x)}$ rule applied to any formula x. Whether this symbol-string does what is claimed, is discussed in the next few paragraphs. The expression uses relations that are defined within $\mathcal{N}$. It is not a member of $\mathcal{L}$.

$$\mathrm{G(x,y)} = (\exists \mathrm{v}((\mathrm{v} < \mathrm{y}) \wedge (\mathrm{ENVbl(v)}) \wedge (\mathrm{y} = 2^3 * 2^3 * 2^{13} * \mathrm{v} * 2^5 * \mathrm{x} * 2^5) \wedge (\mathrm{Gd(x)}))).$$

Although G(x,y) *is written with some symbols used to obtain symbol-strings in* $\mathcal{L}$*, it is an informal string of symbols using members of* $\mathcal{H}$*.* G(x,y) is a relation between Gödel numbers x and y. For the coding in [10, p. 149], y is the Gödel number for the expression $((\forall v)x)$ where v is a Gödel number for a formal variable and x is the Gödel number for a formula from $\mathcal{L}$. The symbol * is technical in nature and its mathematical meaning is not described here. Gödel numbers are generated by a specific algorithm. These numbers can be decoded and sequences of formula can be explicitly displayed. The expression G(x,y) is written this way since one can more efficiently show that it defines a numerical relation in $\mathcal{H}$ that has a special **recursive** property.

How does one know that this symbol-string does match the informal Gen(x, y) statement? By human mental processes, this symbol-string is translated back into an meaningful statement. When this is done, the human brain (mind) of a trained individual will say something like "Yes, I think that the translated statement has the same meaning as the original Gen(y, x)."

Notice that, in general, the symbol-strings to which Gen is applied and its conclusion can vary. The places where the formula appears in a proof can vary. But, the way the coding is done, the Gödel number for formula x to which Gen is applied is, in $\mathcal{H}$, less than the Gödel number for the formula with the additional symbols $(\forall x(\quad))$. Here is my translation, in an equivalent form, for a specific coding [10, p. 149].

> Expressions for the symbols EVbl(v) and Gd(x) are to be inserted into this description. For a Gödel number y, there is a Gödel number v, which represents a formula such that v is less than y, and BVbl(v) and y is the Gödel number for ((∀v)x), and Gd(x).
>
> The EVbl(v) tells us that Gödel number v corresponds to an actual symbol for a variable in $\mathcal{L}$. The Gd(x) states that x is a Gödel number for a formula that appears in the formal theory formally obtained using a **PA** axiom-system. For the above example, v is x_1, and the x is $(\mathrm{x}_1 = \mathrm{x}_1)$.

Gödel's incompleteness theorem utilizes the fact that Gödel numbers, being natural numbers, can be translated into "numbers" in **PA**.

> The idea is not that in informal $\mathcal{N}$ there is a statement that cannot be informally established, but rather Gödel's incompleteness result states that, for **PA**, a certain string of symbols using the informally described methods to write a formal proof cannot appear as a step in a finite list of steps. Recall that the Peano informal definition for induction cannot be formally expressed in $\mathcal{L}$.

Again this says nothing about whether the notion of "truth" can be applied to a $\mathcal{G}$. It is *incorrectly claimed* by many individuals, including a reviewer of Davies' book, that you have the Liar paradox relative to the Gödel's theorem. The Liar paradox occurs as follows: “A man says, ‘What I say is false.’ ” This notion of “false” is a meaningful “false” relative to the meaningful process of “speaking.” This *general* notion can be applied to numerously many different scenarios. The term “false” is not so generally defined within mathematics. What is termed “true” or “false” in mathematics is relative to a few specific actions. The words “true ” or “false” are either terms with no meaning, or they refer to the soon to be defined Aristotle correspondence property for “truth.” For this informal statement, an argument is mounted using the informal meanings for the words between the ‘   ’. When this is done, the language used for the argument and for the statement between ‘   ’ is the same meaningful language. This does not occur for any aspect of a proof for Gödel’s theorem.

Mathematical logic and its methods were first introduced to eliminate contradictions that occur when informal languages are used. And, where it applies, this is accomplished. Gödel’s proof is actually based upon Richard’s 1905 paradox [10, p. 2], as Gödel stated . His methods are devised to circumvent this paradox. All that this theorem states is that you cannot use the informally described procedures to construct a finite formal proof that yields either the $\mathcal{G}$, $(\neg\mathcal{G})$, $\mathcal{R}$ or $(\neg\mathcal{R})$ using **PA**. After the next quotation, these statements are analyzed more carefully using the correspondence property for “truth.”

The following is Kleene’s description [8, p. 256] that shows that there is no contradiction in the informal world.

> “This was the point of departure in Gödel’s original proof, or at least in the heuristic explanation he gave for it; namely, he constructed a formula expressing its own unprovability. This is very close to the paradox of the Liar, where we have a sentence expressing its own falsity. Only now, by Gödel’s substitution of “unprovability” for “falsity,” there is a way out. We wish (and believe in the case of **N** (i.e. **PA**)) that all provable formula are (informally) true. Then, if all true (in a model for **PA**) formulas were provable, we would have “unprovable ≡ (equivalent to) false”, so we would have the paradox of the Liar. The way out now is that not all true formula are provable; in particle $\neg \mathrm{C}_p$ is unprovable though true.

What does Kleene mean by “true”? To discuss this, consider Aristotle’s correspondence property for “truth.”

> $\mathcal{A}$. An *expression* “____” is true if and only if ____ is fact (or occurs etc.). Fact, in the sense of this definition, is strongly demonstrated by observations made by many individuals. Fact also includes meaningful instructions that many individuals can apply using observable entities and basic classical-logic, and that can lead to confirming observations for the expression. The term “many” means all those individuals who comprehend the instructions and who can successfully apply them. The phrase “confirming observations” means that the observed behavior matches the terms and meanings in the expression.‡

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‡ This is the complete Aristotle property for truth. However there are other forms of “truth,” where terms in this definition are qualified.

(Note: Under no circumstances does $\mathcal{A}$ prohibit, due to other factors, one from accepting an expression as describing a fact.) The Kleene "true" is relative to the model-theory defined notion of "true" as discussed below. And, this form of "truth" is an example of $\mathcal{A}$. Further, there are logicians who claim that these Gödel-type statements should not be translated into a common language since these are statements about numbers. However, they can be so translated. It is shown below, under the stated consistency conditions, that any formal statement equivalent to $\mathcal{G}$, $(\neg\mathcal{G})$, $\mathcal{R}$, $(\neg\mathcal{R})$ is "true" with respect to $\mathcal{A}$. For $\mathcal{G}$, this correspondence yields the following informal statement.

> S_1. The expression "we cannot formally prove $\mathcal{G}$" is true since it is a fact that we cannot formally prove $\mathcal{G}$. (Of course, formally prove means that a formal proof cannot be expressed explicitly using an axiom-system for **PA**.)

Notice how the quotation marks are used and how they are removed. Further, S_1 is written in two different languages, our meaningful metalanguage and the meaningless symbol-string $\mathcal{G}$. It is the $\mathcal{G}$ that cannot be formally proved. Thus, there is no contradiction relative to "provability" and this is independent from any interpretation one might give for the sentence $\mathcal{G}$. There is one other claim that Gödel's theorem yields an "infinite regression." This may appear to be so, if one translates $\mathcal{G}$ into an equivalent form and back-and-forth substitution is allowed. But, even in this case, such a regression is merely repeating the same Aristotelian truth.

> S_2 The expression "we cannot formally prove 'we can not formally prove $\mathcal{G}$' " is true since, by S_1, it is fact that we cannot formally prove 'we can not formally prove $\mathcal{G}$'. Apply S_1 to this yields - the expression 'we can not formally prove $\mathcal{G}$' is true since it is fact we can not formally prove $\mathcal{G}$. For the next S_3, parentheses are used rather than quotation marks and substitution is again considered.
>
> S_3 The expression (we cannot formally prove (we cannot formally prove (we cannot formally prove $\mathcal{G}$))) is true since it is fact, from S_2, that we cannot formally prove (we cannot formally prove (we cannot formally prove $\mathcal{G}$)). Then apply S_2. The expression (we cannot formally prove (we cannot formally prove $\mathcal{G}$)) is true since it fact that (we cannot formally prove $\mathcal{G}$). Apply S_1 yields the expression (we cannot formally prove $\mathcal{G}$) is true since it is fact that we cannot formally prove $\mathcal{G}$

There is a reason why the correspondence property for truth holds for these formula. There is a formal proof using **PA** that yields a formal statement that if translated into an informal language is Aristotle's Correspondence Theory of truth for these and many other formal statements provable in **PA** [12, p. 120]. However, there does not exist a single formula that yields $\mathcal{A}$ for all provable statements [12, p. 119].

Some also claim that the proof of Gödel's theorem is invalid since it appears to use the Liar paradox. *This statement is also false.* In the actual proof, two distinct languages are used. First, such statements as $\mathrm{Gen}(x, y)$ can be formally expressed by a formula in $\mathcal{L}$ in a special way so that these formal statements are shown to be provable using **PA**, in particular, using axiom-system **S**. Let the symbol $\vdash_{\mathbf{S}}$ ____ denote the existence of an actual proof for a formula substituted for ____. The $\vdash_{\mathbf{S}}$ is an informal symbol. This symbol has a specific meaning, a set of instructions, described in an informal language forms a finite list of formula where the last one is ____. Unless shown otherwise, the "truth" of this statement follows from $\mathcal{A}$.

There is an informal expression Pf(x,y), which states that "x is a Gödel number for a formal proof using the formal theory **PA**, where the last step $\mathcal{B}_{\mathrm{y}}$ has Gödel number y." Given the Gödel numbers s, t that satisfy Pf(s,t), s can be decoded and each decoded step of the proof can be displayed with decoded $\mathcal{B}_{\mathrm{t}}$ as the last step, and conversely. Hence, the expression "there is a **PA** formal proof with last step $\mathcal{B}_{\mathrm{t}}$" is true if and only it is a fact that there is a formal proof with last step $\mathcal{B}_{\mathrm{t}}$. Recall, that this is the $\mathcal{A}$ requirement.

Expression Pf(x,y) defines a relation between the natural numbers formulaized in $\mathcal{H}$ and this relation has the recursive property. As such, Pf(x,y) is **expressible** by a formula $\mathcal{PF}(x_2, x_1)$. This means that if "Pf(p,g)" is true, in the $\mathcal{A}$ sense, then there is a proof using **S** that has $\mathcal{PF}(p, g)$ as the last step. Therefore, in this case, $\vdash_{\mathbf{S}} \mathcal{PF}(p, g)$. Moreover, if for specific numbers p and g, Pf(p,g) is not true (not a fact), then expressibility requires that $\vdash_{\mathbf{S}} (\neg(\mathcal{PF}(p, g)))$. Consider the formula $(\neg(\exists x_2 \mathcal{PF}(x_2, q)))$. A specific formula for $\mathcal{G}$ is constructed and $\mathcal{G}$ has Gödel number q, and actual formal proofs are constructed such that they yield (1) $\vdash_{\mathbf{S}} (\mathcal{G} \to \neg(\exists x_2 \mathcal{PF}(x_2, q)))$ and (2) $\vdash_{\mathbf{S}} ((\neg(\exists x_2 \mathcal{PF}(x_2, g))) \to \mathcal{G})$. Hence the expressions $\vdash_{\mathbf{S}} (\mathcal{G} \to \neg(\exists x_2 \mathcal{PF}(x_2, q)))$ and $\vdash_{\mathbf{S}} ((\neg(\exists x_2 \mathcal{PF}(x_2, g))) \to \mathcal{G})$ are true via $\mathcal{A}$. When formulas are related by such (1) and (2) statements then they are said to be "logically" equivalent. (Although the following "proof" is an expanded version of most proofs that appear, it may still require careful reading.)

> **The Proof.** The first part of this proof does not require ω-consistency. The axiom-system like **S** is assumed to be consistent. (Of course, if it is not consistent, then any member of $\mathcal{L}$ is provable.) The results presented next are for **S** or for any equivalent axiom-system, and for the formulas discussed in the previous paragraph. For this argument, let axiom-system **S** generate **PA**. In Gödel's proof, assume that $\vdash_{\mathbf{S}} \mathcal{G}$, where $\mathcal{G}$ is a equivalent to $\neg(\exists x_2 \mathcal{PF}(x_2, q)))$. Let r be the Gödel number for the proof $\vdash_{\mathbf{S}} \mathcal{G}$ and q the Gödel number for $\mathcal{G}$. Both r and q are in $\mathcal{H}$. Then, let the informal expression Pf(r,q) have the same meaning as in the last paragraph. This expression also defines a relation ithat has the recursive property.
>
> Under the assumption that $\vdash_{\mathbf{S}} \mathcal{G}$, then, using $\mathcal{A}$, expression "Pf(r,q)" is true since it is a fact a formal proof for $\mathcal{G}$ exists. As above, Pf(r,q) is expressed in **PA** by the formula $\mathcal{PF}(r, q)$, where, as usual, $r,\ q$ are symbols in $\mathcal{L}$ for r and q. The notion of expressibility requires that the informal r and q be related to the formal r and q in that (1) $\vdash_{\mathbf{S}} \mathcal{PF}(r, q)$. In this case, formal logical equivalence and a few additional steps, formally establishes that (2) $\vdash_{\mathbf{S}} (\neg \mathcal{PF}(r, q))$. But, as previously pointed out, this implies that **S** is inconsistent, which contradicts the consistency assumption. So, the assumption that $\vdash_{\mathbf{S}} \mathcal{G}$ is false relative to $\mathcal{A}$. Please note that Pf(r,q) and $\mathcal{PF}(r, q)$ are from different languages.

> (The stronger notion of ω-consistency is not required for the Rosser formula.) The notion of ω-consistency implies that **S** is formally consistent. Using the same approach and ω-consistency and assuming that $\vdash_{\mathbf{S}} (\neg\mathcal{G})$, then, from the equivalence notion, there is a formal proof, where its last step (*) has the form $\exists x_2 \mathcal{PF}(x_2, q)$. The x_2 vary over any of the **S** defined natural numbers. The formula $\mathcal{PF}(x_2, x_1)$ is the above formula that represents Pf(x,y). Since $\vdash_{\mathbf{S}} (\neg\mathcal{G})$, then the formal consistency of **S** implies that it is a fact that a formal proof does not exist that has $\mathcal{G}$ as its last step. Hence, Pf(n,q) is $\mathcal{A}$ false for any natural number n. From the "expressible" notion this means that for any natural n $\vdash_{\mathbf{S}} (\neg(\mathcal{PF}(n, q))$. The definition of ω-consistency states that, in this case, there cannot be a formal proof that has $\exists x_2 \mathcal{PF}(x_2, q)$ as its last step. This contradicts (*). Hence, the assumption that $\vdash_{\mathbf{S}} (\neg\mathcal{G})$ is false. (Note that "the metamathematical reasoning used in the proof of that (this) theorem can be expressed and carried through with **K** (i.e. **PA**) itself [10, p. 166]. (See [6].))

Hence, **PA** is undecidable. This is a meaningful notion applied to meaningless symbol-strings. Thus, this proof does not replicate the requirements of the Liar paradox.

III. Models

To incorporate the notion of "true or false," as displayed by the usual propositional truth-tables, a little set-theory is employed. Using simple aspects of set-theory, objects called **structures** are constructed. These structures lead to the concept of a **model**. Indeed, structures can be constructed from $\mathcal{L}$ itself. Thus, it is hard to deny that these structures exist if you have an actual formal language and you trust the few set-theory notions used.

First, you define or construct a nonempty **domain** D. This is the set of stuff over which any of the translated variable symbols x_1 vary in such statements as "for all x_1" ($\forall \mathrm{x}_1$) and "there exists an x_1" ($\exists \mathrm{x}_1$). For example, the domain D might be the set of symbols $\{0', 0''\}$, where $0'$ and $0''$ are consider as not identical. This set certainly exists since they are but perceivable marks. Relation type symbols in **PA** (predicates), like $\leq$, must correspond exactly to defined objects in the structure. For example, let the formally defined symbol $\leq$ correspond to $\mathrm{R}' = \{(0', 0''), (0'', 0'')\}$. (The R' is a set of two **ordered pairs** as they are ordered from left-to-right.) Denote these objects by $\langle \mathrm{D}, \mathrm{R}' \rangle = \mathcal{M}$. To show that $\mathcal{M}$ **satisfies** formal expressions, the appropriate symbols in a formal expression are translated and it is determined whether the defined objects in $\mathcal{M}$ correspond exactly to the translated expressions.

As an example, consider the formal statement $(\forall x_1 (\exists x_2 (R(x_1, x_2))))$. Translating this into an informal statement for a domain, it states "for each member x_1 in D, there is a member x_2 in D such that x_1 and x_2 are R' related." Unless stated by a formula, the x_1 and x_2 vary independently over D. Let $\mathrm{x}_1 = 0'$. Then it is a fact that for $\mathrm{x}_2 = 0''$, $(0', 0'')$ and $(0'', 0'')$ are in R'. Let $\mathrm{x}_1 = 0''$. Then, for $x_2 = 0''$, $(0'', 0'')$ is in R'. Thus, each member of the domain satisfies R'. The statement that "R' is a model for $(\forall x_1 (\exists x_2 (R(x_1, x_2)))$"

is true by $\mathcal{A}$. Also, one states that $(\forall x(\exists y(R(x,y)))$ "holds" or is "true" for the structure $\langle \mathrm{D}, \mathrm{R}' \rangle = \mathcal{M}$. Notice that $\mathrm{R}'' = \{(0', 0'), (0'', 0'')\}$ is also a model for the statement. This notion of "truth" satisfies the well know ordinary truth-tables and $\mathcal{A}$.

Suppose the same domain is used and that $\mathrm{R}''' = \{(0', 0'')\}$ corresponds to R, then there is no member z of D such that $(0'', \mathrm{z})$ is in R'''. Hence, the structure $\langle \mathrm{D}, \mathrm{R}''' \rangle$ is not a model for $(\forall x_1(\exists x_2(R(x_1, x_2)))$. Hence, $(\forall x(\exists y(R(x,y)))$ does not hold or it is "false" *for the structure* $\langle \mathrm{D}, \mathrm{R}''' \rangle$. The set D need not be composed of symbols from the language $\mathcal{L}$. Sometimes a model is built using symbols that identify members from another mathematical theory. For example, there is an axiom-system that logically yields a philosophic dialectic that requires a potentially infinite domain for a model [WS-6]. To show that this dialectic axiom-system is consistent, $\mathcal{H}$ is used to construct a model.

> *This all means that, for formulas in $\mathcal{L}$, mathematical truth, as defined via models, is equivalent to $\mathcal{A}$ truth.*

There are two very significant metamathematical theorems that relate a formal proof using a set of axioms like **S** with the notion of a model for the axioms.

> (1) Any nonempty set of formal expressions $\mathcal{E}$ taken from $\mathcal{L}$ is consistent if and only if $\mathcal{E}$ has a model [10, p. 71, Prop. 2.17], [WS-4, p. 158, Theorem 3.8.6***].
>
> (2) If, for a nonempty consistent set $\mathcal{E}$ taken from $\mathcal{L}$, a statement F in $\mathcal{L}$ cannot be formally proved from $\mathcal{E}$, then the collection of statements $\mathcal{F}$ that contains and only contains each member of $\mathcal{E}$ and $(\neg F)$ is consistent [10, p.67, Lemma 2.12], [WS-4, p. 156, Theorem 3.8.3]. (Note: Due to how $\mathcal{L}$ is constructed, for any formula E, the symbol-string $(\neg E)$ and all formulas needed to establish all such metatheorems are contained in $\mathcal{L}$.)

The empirically consistent $\mathcal{N}$ can be used to construct a model $\mathcal{M}_N$ for **S**. A major result shows that if **S** has a model, then the structure is a model for each "statement" in **PA**. (What constitutes a "statement" requires a definition since not all members of $\mathcal{L}$ are statements.) In the sequel, another theory that is accepted as consistent also yields a model for **S**. So, if one accepts these ideas, then it can be shown that **S** is ω-consistent. Of course, ω-consistency is not necessary if the Rosser expression is used.

Since $(\neg \mathcal{G})$ cannot be formally prove using a consistent **S**, then consider the set of statements $\mathcal{F}$ that contains only **S** and $\neg(\neg(\mathcal{G}))$. Then $\mathcal{F}$ is consistent, by (2), and has a model $\mathcal{M}_1$, by (1). But, $(\neg(\neg \mathcal{G}))$ is equivalent to $\mathcal{G}$. Thus, $\mathcal{G}$ holds or is true in $\mathcal{M}_1$. The same approach yields that other Gödel type formulas are true in various models. This includes $(\neg \mathcal{G})$. Of course, from the definition of a model, $(\mathcal{G} \wedge (\neg \mathcal{G}))$ does not hold in any of these models. This all assumes that the methods used to construct models are not too controversial.

None of the four Gödel type statements say anything specific about the behavior of the natural numbers. Hence, it is significant, if a statement can be found that does say something about the natural numbers, but, although it is provable, the statement does not have a formal proof using **PA**. This was done in 1977 and is known as the Paris-Harrington Result. It has application to some Davies statements.

IV. The 1977 Paris-Harrington Result.

For the remainder of this article, some very basic set-theory terminology is used. Translate $\in$ as "element in", $\subset$ as "subset," (i.e. $A \subset B$ means the elements of A are elements in B), $A \cap B$ the set of all elements common to A and B, and $A \cup B$ the set of all elements that are in A or B.

1.

> Consider the following nonempty set of six symbols that represent natural numbers. $K_6 = \{0, 1, 2, 3, 4, 5\}$.

2.

> Let K_n denote a set of natural numbers. Let nonempty $H \subset K_n$ and, in general, $\#H$ means the number of elements in H. Let $\#K_n = n$, where $n \geq 1$. For the first part of this example, let $n = 6$ and K_6 the set in 1. Clearly, $1 \leq \#H \leq 6$. For natural number m, where $1 \leq m \leq \#H$, let the "m-graph," $G(m, H)$, be the set that results from writing down all of the **m-element** sets M, where $\#M = m$, that can be constructed from H.

3.

> $G(2, K_6) = \{\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{0,5\},\{1,2\},\{1,3\},\{1,4\},\{1,5\},$
> $\{2,3\},\{2,4\},\{2,5\},\{3,4\},\{3,5\},\{4,5\}\}$.

4.

> A **P-partition** of $G(m, H)$ is a re-grouping of the members of $G(n, H)$ into subsets containing $P \geq 1$ elements. Let $P = 2$. For $G(2, K_6)$, there are 15 2-element sets that are re-grouped into 2 nonempty subsets, called subdivisions. (In set-theory language, a 2-partition of K_6 is composed of two nonempty subsets A, B of K_6 such that $K_6 = A \cup B$ and $A \cap B = \emptyset$ the set containing no elements.) Below are five examples, where I and II denote subdivisions.

5.

> $\mathrm{I} = \big\{\{0,5\},\{0,1\},\{0,3\},\{0,4\},\{2,5\}\big\}$,
> $\mathrm{II} = \big\{\{0,4\},\{1,2\},\{1,3\},\{4,5\},\{1,4\},\{2,3\},\{2,4\}\{3,4\},\{1,5\},\{3,5\}\big\}$.

6.

> $\mathrm{I} = \big\{\{0,1\},\{0,3\},\{1,3\}\big\}$,
> $\mathrm{II} = \big\{$All other 2–element sets.$\big\}$.

7.

> $\mathrm{I} = \big\{\{0,1\},\{1,4\},\{1,3\},\{4,5\},\{3,5\}\big\}$,
> $\mathrm{II} = \big\{\{2,3\},\{2,4\},\{3,4\},\{0,2\},\{0,3\},\{0,4\},\{0,5\},\{1,2\},\{1,5\},$
> $\{2,5\}\big\}$.

8.

$\mathrm{I} = \big\{\{0,1\},\{1,4\},\{1,5\},\{4,5\}\big\}$,
$\big\{\mathrm{II} =$ All other 2–element sets$\big\}$.

9.

$\mathrm{I} = \big\{$All other 2–element sets$\big\}$,
$\mathrm{II} = \big\{\{4,5\}\big\}$.

10.

Notice the following facts about these five 2-partitions. For no. 5, let the set $H = \{3,4,5\}$. Then $\#H = 3$. Observe that $G(2,H) = \{\{4,5\},\{3,4\},\{3,5\}\} \subset \mathrm{II}$.

11.

For no. 6, let $H = \{0,1,3\}$. Then $\#H = 3$. Observe that $G(2,H) \subset \mathrm{I}$.

12.

For no. 7, let $H = \{2,3,4\}$. Then $\#H = 3$. Observe that $G(2,H) \subset \mathrm{II}$.

13.

For no. 8, let $H = \{1,4,5\}$. Then $\#H = 3$. Observe that $G(2,H) \subset \mathrm{I}$.

14.

For no. 9, let (a) $H' = \{0,1,2,3,4\}$. Then $\#H' = 5$. Let (b) $H'' = \{0,1,2,3,\}$. Then $\#H'' = 4$. Let (c) $H''' = \{0,1,2,\}$. Then $\#H''' = 3$. Observe that $G(2,H'),\ G(2,H''),\ G(2,H''')$ are all subsets of partition I. Note that if any $H \subset K_6$ has the property that $\#H > 3$, and $G(2,H)$ is a subset of one of the partitions, then, since 2-element graphs are used, members from set H can be removed, one at a time, to get a set $\underline{H}$ such that $\#\underline{H} = 3$ and the graph $G(2,\underline{H})$ is a subset of the same subdivision.

15.

Now repeat the above process for $K_5 = \{0,1,2,3,4\}$.

16.

$G(2,K_5) = \{\{0,1\},\{0,2\},\{0,3\},\{0,4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},$
$\{3,4\}\}$.

17.

Consider the following 2-partition subdivisions of K_5
$\mathrm{I} = \big\{\{0,1\},\{1,2\},\{0,4\},\{3,4\},\{2,3\}\big\}$,
$\mathrm{II} = \big\{\{0,2\},\{1,3\},\{1,4\},\{0,3\},\{2,4\}\big\}$.

18.

It is not difficult to trace out all of the possibilities like $H = \{0, 1, x\} \subset K_5$ and show that no set $H \subset K_5$ exists such that $\#H = 3$ and such that $G(2, H)$ is a subset of either of these subdivisions. Using the remark in 14, there cannot exist any $H \subset K_5$ such that $3 \leq \#H \leq 5$ and such that $G(2, H)$ is a subset of either of the two subdivisions.

19.

For the, $K_n = \{0, 1, 2, 3, \ldots, n\}$, $n = 5$ or $n = 6$, let P denote the number of subdivisions used. What has been demonstrated above?

20.

For $n = 6$, $P = 2$, $m = 2$ and, as illustrated, for each of the five 2-partition subdivisions, there exists $H \subset K_6$ such that $\#H = 3$, and $G(2, H)$ is a subset of one of the subdivisions.

21.

But, for the case of $n = 5$, we, at least, found one of the possible $P = 2$-partitions, where no such $H \subset K_5$ exists such that $\#H = 3$ and $G(2, H)$ is a subset of either of the subdivisions.

22.

All of the results, thus far, depend only about counting. Consider any distinct six natural numbers $N_6 = \{a_0, a_1, a_2, a_3, a_4, a_5\}$ Then take all the sets previously defined and substitute for 0, a_0, for 1, a_1, for 2, a_2 etc. All that holds, thus far, for K_6 holds for any N_6. Do the same thing for the five element set K_5. Then all such five-element set will also satisfy the analysis in 18.

23.

Why are things like this happening? One version of the Finite Ramsey Theorem, as mentioned in [11], can be formally established using **PA**, and shows that the problem is with n. But, what it says is not necessary for what comes next.

There is one additional observation about the partitions 5 - 9. Let h be the smallest number in H. Notice that, in 10 above, the smallest number $h = 3 = \#H$; in 11, $h = 0 < \#H$; in 12, $h = 2 < \#H$; in 13, $h = 1 < \#H$; in 14, for H''', $h = 0 < \#H'''$. But, for any of the sets constructed in 22, if for the smallest number $k \in N_6$, $k > 6$, then there is no $H \subset N_6$ that can satisfy these inequalities. So, we have an obvious question. Does the following conjecture, on the next page, hold?

> (**) Let $\min(F)$ be the smallest value for the numbers in a nonempty finite set F of natural numbers. For every three positive natural numbers $P,\ m,\ L$, there is a natural number n and a set K_n, where $\#K_n = n$ such that, for every P-partition no matter how the m-element sets are distributed among the P-partition subdivisions, there always exists a nonempty $H \subset K_n$, where $\#H \geq L$, the $\min(H) \leq \#H$ and $G(m, H)$ is a subset of one of the P subdivisions of $G(m, K_n)$.

Can the modern formal set-theory $\mathcal{S}$ be informally showed to be consistent? There does exist a **conceptual** model $\mathcal{M}_S$ that uses of concepts of *platforms where formation takes place, before, after* and *collections composed of formed sets or platforms upon which sets are formed.* As used here, an object can neither be before nor after itself. (The platforms can be considered as made of "glass" if that helps.) There is a platform P' that contains, at least, one "set." For any other platform P, there are certain platforms before P. For each $P \neq P'$, since there are platforms before P, the collection consisting of all sets formed on the platforms before P are formed into sets [In [1, p. 323], the platforms refer to "stages of formation."]. There are no other sets other than those on platforms or formed on a platform. (One could restrict this formation to the sets formed only at the "previous" platform, but this is not necessary.) So, start with a set x on the platform P'. Then you form the set {x}, where "forms" means to gather the previous sets together and put { and } to the left and right. So, at least, the sets you now have include x and {x}. At the next platform, use x and {x}. This yields the forms {x}, {{x}} {x, {x}} and yes there seems to be a repeated form on this platform. Once set-equality is defined, the repeated forms are shown to be equal. The notation y $\in$ x means that y is one of the sets used to form x and y $\notin$ x means that y is not one of the sets used to form x. Notice that sets are actually specially constructed "forms."

The Infinity and Regularity axioms are shown to hold in this model [1, p. 325-327]. Thus, there are a lot of sets with which to work and Regularity means that if y $\in$ x (i.e. x is nonempty), then there is a z $\in$ x such that there is no t $\in$ x such that t $\in$ z (i.e. they have no common members.) Is this significant for the anomalies that made Cantor's set-theory less than desirable? Let v and u be different sets. Is it possible that v $\in$ u and u $\in$ v? Well, assume so. Then consider the set X = {v, u} formed after v and u are formed. (This set exists from the Pairing Axiom and is modeled by $\mathcal{M}_S$.) Then let v $\in$ u, u $\in$ v. Thus, v $\in$ u and u, v $\in$ X, also u $\in$ v and v, u $\in$ X. But, these are the only two sets in X. This contradicts the Regularity Axiom. So, it must be the case that either v $\notin$ u or u $\notin$ v. Which is interesting in itself. Now take any set x and consider the formed set {x, {x}}. Then since x $\in$ {x}, it follows that x $\notin$ x.

Is there a set of all sets? A formal proof shows that no such set exists, but so does $\mathcal{M}_S$. First, if there is a set X of all sets, then it is formed at $P*$. The phrase "all sets" means that X is one of the sets formed before $P*$. Thus, from the formation concept, X $\in$ X. But, the above deduced result that there is no set Y such that Y $\in$ Y holds in $\mathcal{M}_S$. Hence, X is not a set. In showing informally that $\mathcal{S}$ is consistent, it is not really the forming of explicit sets that establishes this, but rather it is the general intuitive meanings for the italicize words in the previous paragraph that establishes consistency. Note that all deduced results using **PA** are members of $\mathcal{S}$.

Paris and Harrington [1, 11] have shown a startling result. They prove conjecture (**). Remarkably, Paris and Harrington show that the **PA** formalized (**) statement cannot be formally established using **PA**. The Finite Ramsey Theorem that can be formally established using **PA** is (**) with the requirement that $\min(H) \leq \#H$ removed.

V. Refutation of Davies' Implications.

First, the most egregious implication of the Davies pronouncements is refuted. It is clear to me that Davies has presented no information that allows one to have any specific knowledge of God as He is described within the Bible and other documents. His assertions imply "[T]hat maybe the ultimate answer cannot be obtained through reason but only through mysticism . . ." [9]. The word mysticism has as its basic ingredient that "ultimate" God-related answers cannot be known through human reasoning. These implications are refuted by the GGU and GID-models and their theological interpretations [3]. If any individual claims that an answer to an ultimate question and an ultimate cause cannot be rationally described, that God cannot be rationally known, that one cannot rationally follow Biblical precepts, that miracles are irrational events and that how the metaphysical world affects our daily existence is an irrational notion, then such claims have been shown to be <u>false</u>. Items in this list can be "rationally" known, where rational is the common everyday classical-logic, the same logical processes used by science-communities. As shown by a GGU-model interpretation, the reason why we cannot have knowledge of the truly <u>deeper</u> aspects of God is that no entity within any physical universe is intelligent enough or has a language that allows for a deeper comprehension.

Davies apparently has no actual experience with the methods God uses to make His supernatural presence known to His created when an implication of his conjecture implies that "Science may offer a surer path to God than Religion" ("God and the New Physics"). The implication that is actual implied in these two his books, is that "may offer" should be replaced with "offers." Davies claims that the universe is intelligently designed but the intelligence is internal to the universe in that it constitutes a complex set of conditions with particular laws of physics based upon the conditions. We recognize this intelligence, but only as it duplicates the "intelligence" displayed by a universal Turing machine. His science has told us that there is *apparent* intelligence in the behavior of our universe. The only mode of rationality he allows is that of the machine and, of course, his restricted notion of what constitutes scientific discourse. His ultimate question is an ontological question, which he claims cannot be established by the only mode of scientific discourse he deems correct. For any such question, he claims imply that his science leads to an answer, which is that the question cannot be relationally answered by methods used to answer physical-science questions.

The ultimate question seems to be, "Is there a reason why are we here?" His pronouncements imply that there is no definite proof that establishes a meaning for our existence. (Of course, using his restricted methods.) Hence, his science dictates that mysticism be employed. His "religious" answer implies that "Maybe we were truly meant to be here." So, any "religious" question that does not lead to a solution using his "reasoned" methods requires mysticism. However, the truly rational statements made in the Bible do answer the question. It is not a "maybe." The Bible explains rationally and exactly why we are here. We are here to form the most significantly part of His everlasting church. A church that will commune with God as very briefly described in Genesis 3:8, but, more vividly illustrated in Revolutions.

What Davies does, as with other individuals who only approach God through the essence of philosophy, is to present unverified **linguistic descriptions** - "paper and pencil" descriptions. Although, I also present such descriptions, I freely admit that without an "indescribable" personal relation with God, one has no means to verify that descriptions for God's attributes are correct. Without such verification, one can expect errors.

Davies' physical science, by definition, cannot describe God's supernatural world. On the other hand, the science of mathematics can, if one considers it a science. It is actually more of an art. But, it is not the mathematics with which Davies is enthralled. He only considers standard everyday mathematics since it, obviously, aids him in expressing his arguments.

It is extremely clear from the Scriptures that the supernatural attributes of God are, at the most, but partially comprehensible when they are compared to physical attributes. For each of these numerous comparisons, it is stated specifically that the supernatural attributes, in power and divinity, are very unlike physical attributes. Scripturally, only through the continuous application of supernatural processes does our universe "continue" to exist. Since standard mathematical structures are used to model physical-system behavior within our universe, using these same structures to model God's supernatural aspects is closely related to pantheism.

The only presently known mathematical approach that avoids the pantheistic correspondence and that does yield, when theologically interpreted, a partial comprehension of God's supernatural attributes as they are compared with of those of His created, is the approach using Nonstandard Modeling. This is the approach used in every article at [WS-5] that analyzes the attributes of a Deity - a Deity with attributes that will always be far superior to any physical-system either within or exterior to our universe (if such exist). These results counter Davies' naive statements that characterize certain Divine attributes of God - the sustaining and continual supernatural control over all aspects of our physical universe - as not rationally describable. (These counters to the Davies' implications are not related to the simplistic computer windows illustration for event sequence construction that appears in [3]. Such a process illustrated in [3] yields additional intuitive comprehension as how event sequences can be pre-designed within the Nonstandard Physical World by a higher-intelligence.)

Next, consider Davies' universal Turing machine conjecture. This conjecture cannot be totally verified. It is a conjecture that requires one to accept that humankind can describe all of the processes that govern the behavior of our universe, and *that such physical processes do not require additional processes that cannot be humanly comprehended OR describable in accepted physical terms.* The conjecture states that:

> *The laws of physics (nature) are "computable" by a universal Turing machine and, hence, the universe lends itself to simulation via such a machine.*

Such machines are idealized computers. Pean Arithmetic determines the computations that any Turning machine can perform. Also they use an infinite tape concept. (Any differences between the notions of potentially infinite and the ordinary infinite are not discussed in this article.) But, basic mathematics used by the physical scientist requires, with certain exceptions, the concept of the *infinite*, at last, as envisioned by the informal natural numbers. For Turing machines, a Gödel type of coding [10, p. 246] is used for any Turning machine language [10, p. 232] such as a language for **PA** and the first-order predicate calculus. A Turning statement is any set of finite strings of alphabet symbols constructed form a finite alphabet. The **PA** natural numbers can be represented for a Turing machine by successions of $|||\ldots$ marks.

One of the problems intuitive when Turing machine concepts are formalized is that the symbolism often gets rather complex. So, for this reason, the symbolism is not introduced, but the intuitive facts are easily understood. Consider the Paris-Harrington result. Let PH be the formal (**) statement as expressed in **PA**, where $\mathbf{PA} \subset \mathcal{S}$. This statement is also formally provable using formal $\mathcal{S}$. Then the additional Paris-Harrington result states that, under the consistency assumption for $\mathcal{S}$, neither PH nor $(\neg PH)$ can be formally proved using **PA**. The reason that $(\neg PH)$ cannot be proved using **PA** is that since $\mathbf{PA} \subset \mathcal{S}$, then this would mean that $\mathcal{S}$ is inconsistent.

All relations such as Pf(x,y) determine a function for all the natural numbers. Let x_1 and y_1 be any natural numbers. Then in a formal proof, if x_1 and y_1 are Gödel numbers and x_1 is the Gödel number for a proof for the expression with Gödel number y_1, then its value is 0. In all other cases, its value is 1. Denote this function by $\mathrm{C_{Pf}}$. Because $\mathrm{C_{Pf}}$ is defined for all pairs of natural numbers, $\mathrm{C_{Pf}}$ is called a **total** function. A relation such as Pf(x,y) is recursive if and only if $\mathrm{C_{Pf}}$ has the recursive property.

Each total function is Turing computable if and only if it has the recursive property [10. p. 249]. A universal Turing machine $\mathcal{U}$ can compute any recursive function [10. p. 255]. Hence, given Pf(x,y), then $\mathrm{C_{Pf}}$ is $\mathcal{U}$-computable. However, if given the Gödel number q for the statement $\mathcal{G}$, then, for any natural number p, $\mathcal{U}$ would compute $\mathrm{C_{Pf}(p,q)} = 1$. Hence, we know that, in translated form, $\mathcal{U}$ cannot supply a proof for $\mathcal{G}$ using any axiom-system equivalent to **S**. I point out that there a formula similar to $\mathcal{G}$, using Turing machine language, which is true in the mathematical sense, but a function that would show that it is or is not provable using **PA** is not Turing computable [9, p. 250, Theorem V]. This leads to a Turing machine "proof" for Gödel's incompleteness theorem. Relative to this result, Kleene states, "To improve the procedure or machine must take ingenuity, something that cannot be built into the machine" [9, p. 246]. (If given a proof, it may be possible for a computer to check whether the proof has been correctly formulated using accepted logical means. One such computer checked proof is Gödel's Incompleteness Theorem.)

It has been established that PH is not provable using **PA**. Statement PH has a Gödel number. But, $\mathcal{U}$, as with q, yields the value 1 for each natural number p for this function.

Thus, as before, $\mathcal{U}$ cannot supply a proof for PH using an axiom-system equivalent to $\mathbf{S}$. However, $\mathcal{U}$ does yield, at least, one 0 for the Finite Ramsey Theorem's formal expression. Translating the Gödel number for this theorem supplies a proof. It is the additional requirement that H has the property that $\min(H) \leq L$ that prevents there being a Gödel number for a proof of PH [9, p. 246-251] although there is a formal proof using $\mathcal{S}$. Is this significant?

> The Paris-Harrington proof that (**) holds uses consistent $\mathcal{S}$ and, as mentioned, the proof can be formalized using the language of $\mathcal{S}$, where PH is the formula at the last step in the proof. This formal proof can be written down and a bunch of papers. How did this written formal proof come about? According to Davies, it came from a physical brain using $\mathcal{U}$-computable physical laws. So, either Davies is in error and there are physical laws that govern the workings of our brains that are not $\mathcal{U}$-computable or the proof is produced by real mental processes that do not correspond to any physical law.

Roger Penrose also concludes that a universal Turing machine cannot reproduce all results of human thought. Penrose's conclusion is related directly to the Halting problem for Turing machines. ("The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics," (1989)).

Is there a possible physical law for inanimate behavior within Davies physical universe that can be modeled directly by (**)? Yes, if one accepts that different parameter values describe different types of universes. For secular physics, it is claimed that the evidence is mounting that our universe is contained in a spatial "something" that is infinite in character "Right now the evidence seems to favor an infinite universe, but it is not yet conclusive" [WS-7]. Another term for this is that such a universe is *open*. "Whether the universe is open or closed does not change with time" [WS-6 Endnote 1]. The notion of open or closed is independent from the notion of "expansion" and whether the expansion is caused by the cosmological constant or dark energy [WS-8]. So, general relativity and the value of a specific parameter, the critical density, would establish that our universe is infinite no matter what else happens to the material within this infinite space. What does this notion of infinite mean?

The mathematics used to predict this physical law is conceptual in character. For example, consider the decimal expansion for the rational number $2/3 = 0.66666\overline{6}$. The symbol $\overline{6}$ means conceptually that we "think" of the 6s as being repeated, and repeated, and repeated without bound as to the number of 6s used. Expressing the entire collection of symbols is possible if the universe also contains an unbound amount of matter. Otherwise, it is but a concept. Shortly, an unbounded universe is discussed with special properties produced by various acceptable parameters. There are various types of universes such as U_1 with an unbound amount of material or, as some claim, the U_2 universe, where the finite material expands into an ever present infinite space. For this article, a U_3 type is the only one considered, where it is only assumed that expansion occurs and it produces a potentially infinite universe. The universe U_3 ideas presented below can be easily modified so as to apply to U_1 and U_2.

For a type $\mathrm{U_3}$ expanding universe, the expansion of the material entities never reverses and the average density of such matter continually decreases. This yields, at least, a potentially infinite space. In this special physical-law produced universe, the notion of the potential infinite space can be described relative to a fixed volume. (A volume measure does depend upon the "metric" used.) Take a box of a fixed volume as measured today in a Earth bound laboratory, where the box is only considered as a space-configuration. Consider any positive natural number n, and disjoint collections $\mathrm{B_p}$ of boxes such that $1 \leq p \leq n$. For example, if n = 4, then there are disjoint collections of 1 box, of 2 boxes, of 3 boxes and of 4 boxes. There are n such collections. For any natural number, eventually, there will be n disjoint collections $\mathrm{B_p}$ contained in the expanding universe. Each collection $\mathrm{B_p}$ takes the place of each natural number p, $1 \leq \mathrm{p} \leq \mathrm{n}$.

The mathematics that Davies finds so exciting and is used to predict that our universe behaves in various ways can be formally expresses using $\mathcal{S}$. This includes the open or closed universe prediction. A law of gravity, Hilbert-Einstein's or Newton's, predicts another accepted physical law - Kepler's Second Law of planetary motion. This law states that a space-configuration, not marked out by physical matter, has the same area over a fixed time-period as a planet ideally moves along its path motion around a gravitating object, like the Sun. Hence, space-configurations do form the conclusion of a physical law and, under the Davies' conjecture, this Kepler result is $\mathcal{U}$-computable.

The critical density is calculated from accepted physical laws and as such determines whether a universe is open or closed. Under Davies' conjecture, this critical density is $\mathcal{U}$-computable. Hence, the open (potentially infinite) universe is a predicted possibility, which, relative to the physical laws used, may be fact. Consider the following 3-dimensional spatial-configurations. Let n be any positive natural number. (Note that for (**), $\mathrm{n} \geq \mathrm{L}$.) There are regions where the $\mathrm{B_p}$ can be considered as single collections or as grouped into 2 or more disjoint $\mathrm{B_p}$ collections at some moment in the evolution of a potentially infinite universe. For example, the natural numbers $\{1,2,3,4\}$ can be collected into singleton sets such that $\{\{1\},\{2\},\{3\},\{4\}\} \leftrightarrow \{\{\mathrm{B_1}\},\{\mathrm{B_2}\},\{\mathrm{B_3}\},\{\mathrm{B_4}\}\}$. The same type of correspondence holds for other collections that contain $(1,2)$, $(2,3)$, where $(1,2) \leftrightarrow (\mathrm{B_1},\mathrm{B_2})$ and $(2,3) \leftrightarrow (\mathrm{B'_2},\mathrm{B_3})\}$, and, in all cases, the $\mathrm{B_2}$ and $\mathrm{B'_2}$ are disjoint. (Notice that the natural numbers used to establish the Pairs-Harrington example and (**) can all be translated by adding 1.)

The statement (**) is a deduced fact about natural numbers using $\mathcal{S}$ and a physical model using the $\mathrm{B_p}$ only depends upon the statement that our universe is potentially infinite. It is straightforward to consider a partition for m-element collections of the $\mathrm{B_p}$ configurations. Hence, although (**) is a true fact about these collections, $\mathcal{U}$ cannot tell us that these configurations exist. Specifically, for a physically produced potentially infinite universe and <u>any</u> positive P, m, L, $\mathcal{U}$ cannot compute an n that shows there is a space-configuration that physically satisfies PH.

As now apparently accepted, it is **dark energy** that is causing the scale-factor to increase via acceleration. That is, the material universe is expanding in an accelerated manner [WS-8]. One of the major theories that allows dark energy to produce this acceleration is called the **quintessence model.** This model is useful since it has parameters that predict many different expansion scenarios. Moreover, it is consistent with a Big Bang styled universe and string-theory. One scenario has our universe expanding until heat-death occurs and leaving, at the least, an enormous number of electrons. Assume that even if, at that time, acceleration has essentially ceased the electrons have greatly exceeded their escape velocity due to the miniscule gravitational affects. Then "distance" between electrons would still continue to expand ad infinitum. The measure of distance, at this time in the universe's development, is very close to the notion of distance that allowed the boxes to be originally constructed in our local environment. In this parameter-controlled possibility, the space-configurations that satisfy (**) can be considered as associated with the elections as our universe evolves. Hence, given any P, m, L, there is an n which satisfies (**) and, eventually, such a space-configuration would have electrons beyond its boundary. This is also so if the volume of the boxes relative to a local measurement taken today is appropriately reduced. This yields an inanimate falsification for Davies' universal Turing machine conjecture. However, the most immediate falsification is relative to mental processes.

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