The GGU-model and Generation of the Developmental Paradigms

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1. Introduction.

The most basic aspects of the General Grand Unification Model (GGU-model) are delineated in [6]. Attempts are made in [2] to illustrate the developmental paradigms associated with event sequences. In this note, the methods demonstrated in [2] are formalized and compared with the developmental paradigms defined formally in [5]. The notion of the "logic-system signature" for a specific logic-system generated scientific theory consequence operator $\mathrm{S_N}$ (in any of its forms) [3] is detailed.

2. Informal Developmental Paradigms.

A nonempty "alphabet" $\mathcal{A}$ is considered as a finite or denumerable set. The informal language L generated from $\mathcal{A}$ [5] has the property that $|\mathrm{L}| = |\bigcup\{\mathcal{A}^n \mid (n \in \mathbb{N}) \wedge (n > 0)\}|$. The denumerable language L can be considered as informally presented and then the following defined sequences embedded as sequences into the G-structure [5], or you can begin with members of $\mathcal{E} = \mathbf{L}$ and construct these developmental paradigms. These developmental paradigms yield the quasi-physical event sequences. The former method is used in what follows. Further, for simplicity, consider a "beginning" frozen segment F [5]. These frozen segments correspond to the notion of a "frozen-frame" in [2].

The idea is to employ the notion of "finite" choice to characterize, at least, partially the developmental paradigms for nonempty countable $\mathrm{D} \subset \mathrm{L}$. Let $\mathbb{N}$ denote the natural numbers. For each $\mathrm{a},\ \mathrm{b} \in \mathbb{N},\ \mathrm{a} \leq \mathrm{b},\ [\mathrm{a}, \mathrm{b}] = \{\mathrm{x} \mid (\mathrm{a} \leq \mathrm{x} \leq \mathrm{b}) \wedge (\mathrm{x} \in \mathbb{N})\}$ and, as usual, symbol $\mathrm{f}|\mathrm{A}$ denotes the restriction of the function f to a subset A of its domain.

Definition 2.1 (1) For $1 \in \mathbb{N}$, let $\mathcal{Y}_1 = \{\mathrm{f} \in \mathrm{D}^{[0,1]} \mid (\mathrm{f}(0) = \mathrm{F}) \wedge (\mathrm{f}(1) = \mathrm{x} \in \mathrm{D})\}$.

(2) Assume that $\mathcal{Y}_\mathrm{n}$ has been defined. Let $\mathcal{Y}_{\mathrm{n}+1} = \{\mathrm{f} \in \mathrm{D}^{[0,\mathrm{n}+1]} \mid (\mathrm{f}|[0,\mathrm{n}] \in \mathcal{Y}_\mathrm{n}) \wedge (\mathrm{f}(\mathrm{n}+1) = \mathrm{x} \in \mathrm{D})\}$.

(3) Define $\mathcal{Y} = \bigcup\{\mathcal{Y}_\mathrm{n} \mid \mathrm{n} \in \mathbb{N}\}$.

(4) The actual developmental paradigms DP for a particular F are $\mathrm{DP} = \{\mathrm{f} \in \mathrm{D}^\mathrm{A} \mid (\mathrm{A} = \mathbb{N}) \wedge (\mathrm{f}(0) = \mathrm{F}) \wedge (\forall \mathrm{n}((\mathrm{n} > 0) \wedge (\mathrm{n} \in \mathrm{A}) \to (\mathrm{f}|[0,\mathrm{n}] \in \mathcal{Y}\}$. Note: For $\mathrm{A} = \mathbb{N},\ \mathrm{n} > 0$, if $\mathrm{f} \in \mathrm{D}^\mathrm{A}$, then $\mathrm{f}|[0,\mathrm{n}] \in \mathcal{Y}$ iff $\mathrm{f}|[0,\mathrm{n}] \in \mathcal{Y}_\mathrm{n}$.

All of this is extended to the hyperfinite when embedded into the nonstandard structure. In [2], a "master" event sequence is used in an attempt to model Definition 2.1 in a reasonably comprehensible manner using various constructive illustrations. Notice that such a master event sequence is a member of DP under Definition 2.1. Obviously, Definition 2.1 is not the only way to obtain developmental paradigms. Indeed, a simple induction proof shows that $\{f \in D^A \mid (A = \mathbb{N}) \wedge (f(0) = F)\} = DP$. Clearly, $DP \subset \{f \in D^A \mid (A = \mathbb{N}) \wedge (f(0) = F)\}$. Let $d \in \{f \in D^A \mid (A = \mathbb{N}) \wedge (f(0) = F)\}$. Then $d(0) = F$, and $d(1) \in D$ imply $d|[0,1] \in D^{[0,1]}$ and $d|[0,1] \in \mathcal{Y}_1$. Suppose that $d|[0,n] \in \mathcal{Y}_n$. Then $d|[0,n+1]$ means that $d|[0,n]$ and $d|n+1$ and $d(n+1) \in D$. Thus, since $d|[0,n+1] \in D^{[0,n+1]}$, $d(0) = F$, $d|[0,n] \in \mathcal{Y}_n$, it follows that $d|[0,n+1] \in \mathcal{Y}_{n+1}$. Therefore, by induction for each $n > 0$, $n \in \mathbb{N}$, $d|[0,n] \in \mathcal{Y}_n$, and $d(0) = F$. Hence, $d \in DP$. If Definition 2.1 is restricted to the "potential" infinite, then step (3) and (4) are not included. It is step (4) that some might consider as requiring the Axiom of Choice in that members of each $\mathcal{Y}_n$ that are not completely specified are employed in the set-theoretic definition. This somewhat constructive way to define the DP is used to indicate that, at the least, major portions of the basic definition can be obtained via finite choice. This type of finite characterization, when extended to the NSP world, allows for further interesting observations. For example, the event hypersequences can have ultranatural events not merely associated with nonstandard primitive time but also can have them at standard moments of primitive time. This additional property has not been discussed in [2]. Moreover, this possibility also leads to ultrawords and ultimate ultrawords to which ultralogics can be applied since such developmental paradigms can be considered as of the d' type discussed in section 9.1 in [5].

3. Refined Developmental Paradigms.

The illustrations in [2] for generating event sequences using Definition 2.1 do not correspond to the actual technical definition that appears in Chapter 7 in [5] except under a specific restriction. (Note: The usual structure now employed for what follows is the Extended Grundlegend Structure (EGS) $\mathcal{Y}_1$ as defined in [5, p. 70. 82]. The ground set for the standard superstructure is the set of atoms $A_1 \cup A$, $A_1 \cap A = \emptyset$, where A_1 is isomorphic to the natural numbers and A is isomorphic to the real numbers. The set A is usually denoted by $\mathbb{R}$.) Definition 2.1 and these illustrations if restriction to a small primitive time interval $[a, b)$ do correspond to those used in [5]. In that case, the actual complete developmental paradigm would be a countable collection of such developmental paradigms for each $[a, b)$. This would technically require that, for applications such as discussed in [2], an additional collection of ultimate ultrawords be considered via Theorem 7.3.4 in [5]. However, the complete developmental paradigm as the denumerable union of denumerably many sets can also be considered as a denumerable sequence in primitive "time" when the Axiom of Choice is assumed. In this case, the complete developmental paradigm can be generated by a basic ultraword and the ultralogic ${}^*\mathbf{S}$. The method devised in Chapter 7 of [5] to analyze a developmental paradigm is significant and should be used since it yields the greatest control and, in the EGS, displays the ultranatural events. Requiring that the denumerable union of denumerably many objects be denumerable is not necessary if the notion of the developmental paradigm is simply defined via different denumerable sets of primitive identifiers. These notions are now formalized within the standard EGS.

It can be assumed that what follows is the result of an embedding into the standard superstructure of the informal objects. Let $\mathbf{Z}$ denote the integers and consider $\mathbf{Z} \times \mathbb{N}$.

Definition 3.1 For each $(i,j),\ (p,m) \in \mathbf{Z} \times \mathbb{N}$, where $(a,b) = \{\{a\},\{a,b\}\}$, let $\preceq$ be defined as follows:

(1) if $i < p$, then $(i,j) \prec (p,m)$.

(2) If $i = p$ and $j < m$, then $(i,j) \prec (p,m)$, and

(3) $(i,j) = (p,m)$ if and only if $i = p$ and $j = m$.

The binary relation $\preceq$ yields a simple order in $\mathbf{Z} \times \mathbb{N}$.

The intervals $[a,b)$ employed in [5, p. 61] may be replaced with the following specifically defined intervals. Consider nonzero $K \in \mathbb{N}$. When (i,j) appears as a subscript, it is often written as ij. For each $i \in \mathbf{Z}$, let $c_i = i/K$. For the rational numbers $\mathbf{Q}$ and each $i \in \mathbf{Z}$, let $[c_i, c_{i+1}) = \{x \mid (x \in \mathbf{Q}) \land (c_i \leq x < c_{i+1})\}$. For such $i \in \mathbf{Z}$, partition $[c_i, c_{i+1})$, in the same manner as done in [5, p. 61], by a denumerable increasing sequence of partition points $t_{ij},\ j \in \mathbb{N}$, such that $t_{i0} = c_i$, $t_{ij} \in [c_i, c_{i+1})$ and $\lim_{j\to\infty} t_{ij} \to c_{i+1}$. For various $i \in \mathbf{Z}$, the set of rational numbers $\{t_{ij} \mid j \in \mathbb{N}\}$ models a "primitive (time) interval." For example, let $t_{ij} = (1/K)(i + 1 - 1/2^j)$. For applications, one might employ a finite sequence $\{[0, c_1) \ldots, [c_j, c_{j+1})\}$ of such intervals or partition $[0, +\infty)$, $(-\infty, 0)$, or $(-\infty, +\infty)$ using collections of such intervals. If the collection of primitive intervals is nonempty and finite, then there are denumerably many partition points. If the collection of primitive intervals is infinite, then, using the Axiom of Choice, there are denumerably many partition points. If $t_{ij},\ t_{pm}$ are any of these constructed partition points, then $t_{ij} \leq t_{pm}$ (the standard rational number simple order) if and only if $(i,j) \preceq (p,m)$.

By construction, $r \in \mathbf{Q}$ is a partition point if and only if r corresponds to a frozen segment $\mathbf{F}_r$. A major aspect associated with applications is the difference between primitive and observer time. All of the results in [5] that deal with developmental paradigms are relative to countably many collections of frozen segments. Although for certain applications the actual physical events may be repeated, the construction of the developmental paradigm allows $\mathbb{N}$ to be mapped bijectively onto $[a,b)$. Only intervals of the form $[a,b)$ are considered in [5]. Then, for a countable collection of such partitioned intervals, there is an ultimate ultraword that generates, for each $[a,b)$ interval, the appropriate ultraword from which each interval's developmental paradigm is obtained. For this refined approach, the use of the $\mathbb{N}$ notation can be retained under the view that there is a bijection from the set of all partition points onto $\mathbb{N}$. However, it is a rather trivial matter to re-express each developmental paradigm and its standard frozen segments in terms of the appropriate denumerable subsets of $\mathbf{Z} \times \mathbb{N}$ that correspond to the partition points t_{ij}. If this correspondence is employed, then each frozen segment $\mathbf{F}_{ij}$ corresponds to the partition point t_{ij}. The order $\leq_d$ defined on a developmental paradigm $\mathbf{d}$ is the simple order induced by $\preceq$ when the developmental paradigm is properly defined. The "equality" $=_d$ is set equality. The use of this refined partition point notion yields certain more detailed characteristics for event sequence behavior for the General Grand Unification model (GGU-model) [2,6].

As examples of the refined use, as in [5] let only members of $[c_i, c_{i+1})$ be considered. Then an embedded standard developmental paradigm $\mathbf{d}$ is sequentially presented by considering a defining bijection $f\colon(\{i\} \times \mathbb{N}) \to \mathbf{d}$. (Bijections such as this model how members of $\mathbf{d}$ may be "grouped together.") Hence, for any $j \in \mathbb{N}$, and any $p \in \mathbb{N}$, such that $j < p$, $f(i,j) <_d f(i,p)$ and $f(i,p) \neq f(i,j)$. Using EGS and *-transfer, this yields that for $\nu \in {}^*\mathbb{N} - \mathbb{N}$ and each $n \in \mathbb{N}$, ${}^*f(i,n) = f(i,n) \neq {}^*f(i,\nu)$, $f(i,n) \, {}^*{<_d} \, {}^*f(i,\nu)$, and ${}^*f(i,\nu) \in {}^*\mathbf{d} - \mathbf{d}$. In this application, such objects as ${}^*f(i,\nu)$ are called ultranatural events. Consider a partition of $(-\infty, 0)$ and assume that the developmental paradigm $\mathbf{d}$ is determined by a bijection $f\colon(\mathbf{Z}^{<0} \times \mathbb{N}) \to \mathbf{d}$. Let $i \in \mathbf{Z}^{<0}$. Then for any $j \in \mathbf{Z}^{<0}$ such that $i < j$, $f(i,n) <_d f(j,n)$, $f(i,n) \neq f(j,n)$ for each $n \in \mathbb{N}$. By *-transfer, let $i = \nu \in {}^*\mathbf{Z}^{<0} - \mathbf{Z}^{<0}$. It follows that ${}^*f(\nu,n) \, {}^*{<_d} \, {}^*f(j,n)$, ${}^*f(\nu,n) \neq {}^*f(j,n)$ for each $j \in \mathbf{Z}^{<0}$ and $n \in {}^*\mathbb{N}$. Hence, for each $n \in {}^*\mathbb{N}$, ${}^*f(\nu,n) \in {}^*\mathbf{d} - \mathbf{d}$ since $\mathbb{N} \subset {}^*\mathbb{N}$. Each of the ${}^*f(\nu,n)$ members of ${}^*\mathbf{d} - \mathbf{d}$ is called an *initial member*. Let $\lambda \in {}^*\mathbb{N} - \mathbb{N}$. Note that $\mathbf{d} \subset d_2 = \{\, {}^*f(x,y) \mid (x \in {}^*\mathbf{Z}^{<0}) \wedge (\nu \leq x) \wedge (y \in {}^*\mathbb{N}) \wedge (y \leq \lambda)\} \subset {}^*\mathbf{d}$ and d_2 is hyperfinite. In general, the set of all initial members of ${}^*\mathbf{d} - \mathbf{d}$ form an external subset of ${}^*\mathbf{d}$, and, for certain applications, one can consider all such initial members as corresponding to the same *description. For the case of the partitioning of $[0, +\infty)$, the same analysis yields members of ${}^*\mathbf{d} - \mathbf{d}$ that are termed the *final members* in ${}^*\mathbf{d}$.

In section 10.2 of [5], the hyperfinite choice operator is discussed. This is easily extended to the "hyperfinite ordered choice" operator for internal subsets of ${}^*\mathbb{N}$. By considering any nonempty finite $\mathbf{F} \subset \mathbf{d}$, induction shows there is a $\leq_d$ largest member in $\mathbf{F}$. Using this fact, induction shows that for any such finite subset $\mathbf{F}$ there exists a finite choice function that can be considered as arranging members of $\mathbf{F}$ in the proper $\leq_p$ order. The results of applying ${}^*\mathbf{S}$ to the appropriate ultrawords yields a hyperfinite set d'_1 that contains the embedded developmental paradigm $\mathbf{d}$ for interval $[a, b)$ or for intervals such as $(-\infty, 0)$, $(-\infty, +\infty)$, $[0, +\infty)$. Theorems such as 10.1.1 in [5] are independent from the actual type of partitioning used. They only employ the fact that $\mathbf{d}$ is denumerable. Using the partition ordering on $\mathbf{d}$, where members of $\mathbf{d}$ are considered as members of the any of the four primitive interval, the ordering can be restored either by applying ${}^*f^{-1}$ to the internal d'_1 or by considering the general hyperfinite ordered choice function (operator). Under this view, the operator ${}^*\mathbf{S}$ is composed with either of these two hyperfunctions and this yields all the members of d'_1 and, hence, $\mathbf{d}$ in their assigned order.

4. Logic-System Signatures.

In formal logic, a certain amount of mental activity must be done before a formal proof is presented. For example, in most cases of interest, one needs to select finitely many well-formed formulas (wwfs) from potentially-infinite collections of wwfs. This is an acceptable process as modeled by a finite choice function. Further, such things as whether a variable is free or bound may need to be determined and when generalization is appropriate. Of course, there is also the mental activity required just to represent a collection of symbols in the proper form. When a formal deduction is presented, none of this mental activity is presented, although it might be discussed in an external manner using a metalanguage. Thus, not exhibiting such mental activity in the final product is a basic mathematical approach. In what follows, such external mental activity is also required and not represented in the final results.

For a given nonempty language L, science-community scientific theories are discussed in [3]. One considers an implicit or explicit general rules of reference $\mathbf{RI}(\mathbf{L})$ that generates a finite consequence operator $\mathrm{S_N}$ that represents a particular scientific theory. For each $\mathrm{X} \subset \mathrm{L}$, even with the realism relation $\mathrm{R(X)} = \mathrm{S_N(X)} - \mathrm{X}$ applied, there is a vast amount of extraneous "deduction" where the deduced members of $\mathrm{S_N(X)}$ are used to obtain the actual "descriptions, words or images" $\mathrm{P_X} \subset \mathrm{S_N(X)}$ [2,6] that can be perceived. The term "perceived" often means "to become aware of, through application of a set of defined human or machine sensory apparatus."

The actual set $\mathrm{P} \subset \mathrm{L}$ *that constitutes what is termed here as "perceived" should* *be explicitly defined by a science-community for a specific scientific theory.* Hence, (A) for each $\mathrm{X} \subset \mathrm{L}$, an acceptable choice function is applied to $\mathrm{S_N(X)}$ in order to obtain a science-community defined *unique* $\mathrm{P_X} \subset \mathrm{P} \cap \mathrm{S_N(X)}$. Further, if a statistical statement is included that implies that members of $\mathrm{P_X}$ only have a certain probability of being perceived, then the ultralogic investigated in [4] is coupled with the $\mathrm{P_X}$ images. A "signature" is an entity that signifies the presence of a specific process or object.

The notion of the **J**-relation as defined in [3] is now modified. Certain members of X may need to be tagged if they are also members of $\mathrm{P_X}$. These members of X are considered as not being altered by the physical processes involved. The modified $\mathbf{J}'$-binary relation behaves like an identity relation for members of X except that the second coordinate is the same as the first coordinate with one additional fixed symbol attached to each member and the symbol does not appear in any of the perceived members of L. This symbol would not affect the actual "meaning" of any perceived member of L except that the symbol indicates that no change has been made in the expression denoted by the symbol by the physical processes being modeled.

For a given nonempty $\mathrm{X} \subset \mathrm{L}$, a general rules of inference $\mathbf{R_X}$, an "$\mathbf{R_X}$-signature" (a "behavior-signature") and a general rules of inference $\mathbf{RI(L)_S}$, the "$\mathbf{RI(L)_S}$-signature" (the "theory-signature") are determined by $\mathbf{RI(L)}$. Note: In many of these investigations, the customary notation for "n-tuples" is employed where the actual definition may require the more formal definition by the ordered pair concept and induction or functions defined on various $[1, n],\ n > 0,\ n \in \mathbb{N}$.

Definition 4.1 Given L, a general rules of reference $\mathbf{RI(L)}$, any nonempty finite $\{\mathrm{x_1}, \ldots, \mathrm{x_n}\} = \mathrm{X} \subset \mathrm{L}$ and $\mathrm{P_X} \subset \mathrm{P}$ determined by (A). If $\mathrm{P_X} \neq \emptyset$, define $\mathbf{R_X} = \{(\mathrm{x_1}, \ldots, \mathrm{x_n}, \mathrm{x_{n+1}}) \mid \mathrm{x_{n+1}} \in \mathrm{P_X}\}$. Define the $\mathbf{RI(L)_S}$-signature to be the unification $\mathbf{RI(L)_S} = \bigcup\{\mathrm{R_X} \mid (\emptyset \neq \mathrm{X} \in \mathcal{F}(\mathrm{L})\}$, where $\mathcal{F}$ is the finite power set operator.

For a given X, the behavior-signature determines a consequence operator $\mathrm{S_{N_X}}$ that is weaker than $\mathrm{S_N}$, after application of the realism relation (i.e. $\mathrm{S_{N_X}(X)} - \mathrm{X}$) extraneous deductions as well as the actual physical laws descriptions can be eliminated. It is a rather significant GGU-model consequence operator especially relative to the GID-model interpretation. The finite consequence operator $\mathrm{S_{N_S}}$ generated by $\mathbf{RI(L)_S}$ is stronger than each $\mathrm{S_{N_X}}$. By considering the proof of the first part of Theorem 2.4 in [1], it follows that $\mathrm{S_{N_S}}$ is weaker than $\mathrm{S_N}$. In either of these signature cases, after the realism relation is applied, if needed, what is perceived may be further controlled by the ultralogic [4] that generates the statistical observations.

There is a type of converse to Definition 4.1. Rather than starting with the S_N, one can consider selecting for each $X \subset L$, any nonempty $P_X \subset L$. The selected P_X can, of course, be considered as "perceived" or "observed." Using each selected P_X, a $\mathbf{R_X}$ is defined. One considers a "unification" $\bigcup\{R_X \mid (\emptyset \neq X \in \mathcal{F}(L)\} = \mathbf{RI}$. However, due to the general logic-system algorithm, even if one considers the finite logic-systems $\mathbf{R_X}$ as separately applied, there are examples where the results need not be the same as those obtain by application of **RI**. This fact can have significance for empirical science, where only such behavior-signatures are used to establish a rational theory S_N based upon a single physical law. One approach to correct this problem is to analyze carefully the data produced, alter how the data are expressed and produce a collection of behavior-signatures that do correspond to those obtained from the corresponding **RI**. In this case, the **RI** can be consider as a representation for a physical law. Of course, these signature ideas can be applied to other appropriate "natural" laws that may not be considered as satisfying the strict definition for what constitutes a physical law.

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