Nonstandard Analysis Applied to Special and General Relativity - The Theory of Infinitesimal Light-Clocks

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CONTENTS

Preface

It is actually dangerous for me to present the material that appears within this book due to the usual misunderstandings. Any scientist who claims that there are fundamental errors within the foundational methods used to obtain Einstein's General and Special Theories of relativity may be greatly ridiculed by his colleagues who do not read carefully. The reason for this has nothing to do with science but has everything to do with scientific careers, research grants and the like. Thousands upon thousands of individuals have built their entire professional careers upon these two theories and their ramifications. The theoretical science produced is claimed to be "rational" since it follows the patterns of a mathematical structure. As a mathematician who produces such structures, it is particular abhorrent to the scientific community if I make such a claim. Mathematicians seem to have an unsettling effect upon some members of the physical science community, especially when a mathematician delves into a natural science. After all, it was the mathematician Hilbert who, without any great effort, was actually the first to present, in a public form, the so-called Einstein gravitational field equations.

Now please read the following very carefully. The results presented here and in my published papers on this subject are not intended to denigrate those scientists who have, in the past, contributed to these Einsteinian theories or who continue to do so. The corrections I have made are in the foundations for these theories. The corrections are totally related to how the results are interpreted physically. These corrections do not contradict the results obtained when the Einsteinian approach and the language used are considered as **models** for behavior. These corrections are based upon newly discovered rules for rigorous infinitesimal modeling. These results may also be significant to those that hold to the belief that many events within the natural world are produced classically by a zero-point radiation field.

Many unqualified individuals continue to present their own alternatives to these Einstein theories, some claiming that the results are but an exercise in high-school algebra. Certain scientific groups tend to categorize any and all criticisms of the Einstein theories as coming from the unqualified and lump such criticisms into the same unworthy category. However, many highly qualified scientists of the past such as Builder, Fock, Ives and Dingle have made such claims relative to the foundations of these two theories. For Ives, the fundamental approach was to assume that such a thing as length contraction, and not time dilation, is a real natural effect and it is this that leads to the Einstein conclusions. In order to eliminate these criticisms, Lawden states the "modern" interpretation that length contraction has no physical meaning, and only "time dilation" is of significance. This modern assumption is certainly rather ad hoc in character. Further, many theory paradoxes still appear within the literature and are simply ignored by the scientific community. There is, however, a reason for this.

The actual approach used can now be shown explicitly to contain logical error. It was not possible to show this until many years after the theory was fully developed. Further, the original approach utilizes a "geometric language," a language that has been criticized by many including John Wheeler as the incorrect approach to analyze the fundamental behavior of universe in which we dwell. Although Einstein used an explicit operational approach in his Special Theory, he was unable to use a mathematical approach that encapsules his operational definition since the actual mathematics was not discovered until 1961. He used what was available to him at the time. In this book, all such errors and paradoxes are removed by use of the modern corrected theory of infinitesimals and infinite numbers as discovered by Robinson. Moreover, the recently discovered correct rules for infinitesimal modeling are used, and this eliminates the need for tensor analysis and

Riemannian geometry.

The logical errors occur when rigorous mathematical procedures are applied but the abstract mathematical theory uses modeling procedures (i.e. interpretations) that specifically contradict mathematical modeling requirements. These errors are detailed within this book. Unfortunately, the same confused approach often pervades most "physical" interpretations for mathematical structures.

As mentioned, in the articles contained in this book, these errors are eliminated by application of the corrected theory of infinitesimals as discovered by Abraham Robinson. However, in so doing, significant differences in "philosophy" appear necessary. These differences are amply discussed in part 1 of the first article. Note that each article begins with an extensive abstract the contents of which I will not reproduce in this preface.

The results in this book eliminate all of the known controversies associated with these two theories. Indirectly, the results show the logical existence of a privileged frame of reference. From very elementary assumptions gleaned from laboratory observation, it is shown that there is neither absolute time dilation nor length contraction, but there is an alteration in one and only one mode of time and length measurement due to relative velocity (i.e. velocity), potential velocities, textual expansion and the like. These alterations have a "physical" cause. The apparent alteration of mass, the gravitational redshift, the transverse Doppler effect and other verified consequences of these two theories are predicted and shown to have "physical" causes – causes associated with a nonstandard substratum field.

The order in which these articles appear in this book is somewhat reversed from the order in which they were written. Article 2, *A Corrected Derivation for the Special Theory of Relativity*, was written first and presented first before the Mathematical Association of America. Article 1 and Article 3, *Foundations and Corrections to Einstein's Special and General Theories of Relativity, Article 2 and Article 3*, were written from November 1992 – Sept. 1993. However, Articles 1 and 3 contain, almost exclusively, classical mathematics, (with minor exceptions) and are more easily comprehended by scientists versed in this subject. Article 2 requires the additional concepts associated with elementary nonstandard analysis. Article 3 can be comprehended relative to the results presented. The necessary formal infinitesimal theory presented in Article 3 should not detract from this comprehension. The material in article 3 was partially funded by a research grant from the U. S. Naval Academy Research Council. [Added May 2001. Listed below and elsewhere are four published journal articles and a few archived references relative to the methods presented within this book. These references update some of the reference information listed at the end of each article.]

It is hoped that the conclusions developed throughout these articles that are ultimately dependent upon the concepts of nonstandard analysis will motivate the scientific community to become more conversant with proper infinitesimal modeling techniques.

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Special and General Theories of Relativity, Article 1.

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Abstract: In article 1 of this paper, newly identified logical errors in the derivations that yield Einstein's Special and General Theories of Relativity are discussed. These errors are much more significant than those identified by Fock. The basic philosophy of science used as a foundation for these theories is identified. The philosophy of the privileged observer is detailed. Article 1 concludes with a brief discussion of a new derivation for the Lorentz transformation that eliminates all the logical errors associated with previous derivations as well as eliminating the controversial concepts of "length contraction" and "time dilation."

1. Introduction

Nobel prize winner Max Planck (1932, p. 2) wrote:

> Nature does not allow herself to be exhaustively expressed in human thought.

The D-world mathematical model as discussed by this author (Herrmann 1990, p. 12) is used to analyze the linguistic methods used by modern science and develops the following rational possibility.

> Human beings do not have the ability to comprehend and will not eventually describe in human languages all of the true laws or natural events that govern the cosmos. This includes the laws or natural events that govern the development of individual natural systems.

Hence, the philosophy of science as espoused by Planck, and denoted by (A), can be logically argued for by using mathematical reasoning.

On the other hand, the philosophy of scientism, which is denoted by (S), assumes the negative of Plank's statement. One abiding rule of scientism is that nature will allow herself to be exhaustively expressed in human thought. Nowhere in modern science do these two contradictory philosophies of science clash more violently than in the æther or medium concept associated with electromagnetic propagation. Our concern in the first sections of this paper, will be the æther concept, a concept that is partially discussed in numerous journal papers. (See for example Benton, 1988).

2. Some Ether History

Newton attempted to imagine properties of a medium for his universal theory of gravity for in his letters to Boyle he apparently stated that:

> . . . he found he was unable, from experiment and observation, to give a satisfactory account of this medium and the manner of its operation in producing the chief phenomena of nature (Maxwell 1965b, p. 487).

Maxwell (1965b, p. 764) tells us that the only medium that survived as a structure and that seems to uphold the propagation of light was that proposed by Huygens. It was Thomson (1854) who did most of the calculations as to the mechanical properties that such a "luminiferous æther" should possess. Then, in 1864, Maxwell (1965a, pp. 526 – 597) outlined the general properties he would imagine to hold for an electromagnetic field and proposed that the luminiferous æther and the electromagnetic medium are the same.

The mechanical properties of such an electromagnetic æther needed to be expressed in a scientific language and required the basic methods of mathematical deduction. Apparently, most of the believers in such an æther were following philosophy (S) with the additional requirement of *absolute realism.* Absolute realism, in this case, means that an individual corresponds specific physical terms to a list of selected abstract mathematical terms and for a "physical theory" to be accepted those physical entities being named by the physical terms are assumed to exist in reality. Unfortunately, under the usual correspondence, various æther calculations produce physical behavior that seemingly can not exist within our universe. One such difficulty was relative to æther stresses.

Lorentz (1952, p. 31) proposed that the concept of realism be altered.

> I should add that, while denying the real existence of æther stresses, we can still avail ourselves of all of the mathematical transformations by which the application of the formula (43) may be made easier. We need not refrain from reducing the force to a surface-integral, and for convenience's sake we may continue to apply to the quantities occurring in this integral the name stresses. Only, we must be aware that they are only imaginary ones, nothing else than auxiliary mathematical quantities.

However, altering realism in this manner would slightly remove human deductive logic, as it is encapsulated within mathematical reasoning, as a bases for (S). The burning question would be why the human mind needs these "imaginary" entities to properly describe the behavior of the æther? Under the (S) philosophy, either the æther did not correspond to reality or there would need to be new concepts developed and corresponding physical terms defined. But the situation is more complex than this since realism always depends upon a theory of correspondence.

3. The Calculus

Newton's concept of mathematical modeling was firmly rooted in the natural world.

> Geometry does not teach us to draw lines, but requires them to be drawn, for it requires that the learner should first be taught to describe these accurately before he enters geometry, then it shows how by these operations problems may be solved. To describe right lines and circles are problems, but not geometrical problems. The solution of these problems is required from mechanics, ... therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measure (Newton, 1934 p. xvii).

Newton's claim is that our observations and intuitive comprehension of mechanics comes first and then these concepts are abstracted to include the vague notion that objects have certain "capacities or potentials to do things." We are told that it is *after* experimentation, observation and reflection that the mathematical structure is evoked and these "easy" capacity concepts are modeled.

Newton used the language of *infinitesimals* within all of his applied mathematics. To him, these infinitesimal quantities existed in objective reality, they referred to measures of actual natural world behavior. For example, in 1686, Newton explains to his critics what he claims is the easily comprehended notion of the *ultimate velocity*, or what we now term as *instantaneous velocity*, for an actual real material object.

> But by the same argument it may be alleged that a body arriving at a certain place, and there stopping, has no ultimate velocity; because the velocity, before the body comes to the place, is not its ultimate velocity; when it has arrived, there is none. But the answer is easy; for by the ultimate velocity is meant that with which the body is moved, neither before it arrives at its last place and the motion ceases,

> nor after, but at the very instant it arrives; that is, the velocity with which the body arrives at its last place, and with which the motion ceases (Newton, 1934, pp. 38–39).

The abstract notion of instantaneous velocity may have been "easy" for Newton to grasp, but it was incomprehensible to Berkeley and many others who believed that such abstractions could not be applied to actual real material objects. The paramount philosophy of science for Berkeley was a science of the material and directly observed universe. Any arguments that relied upon such abstractions would need to be rejected.

Newton's approach created a schism in the philosophy of mathematical modeling. One group of scientists believed that there exists actual real world entities that can be characterized in terms of infinitesimal measures of time, mass, volume, charge, and the like. Another group assumed that such terms are auxiliary in character and do not correspond to objective reality.

In the early 19th century, Cauchy (1821) using the language of infinitesimals established a result that Able (1826) showed by counter-example could not be logically correct. However, no direct error can be found in the Cauchy's logical argument. Hence the intuitive assumptions underlying the behavior of Newton's infinitesimals must be logically contradictory. Unfortunately, the vast majority of the scientific community still use Newton's infinitesimal concepts. How many know that such a use can lead to logical contradictions?

A few years ago, Robinson (1966) removed the contradictions from the theory of infinitesimals. The new corrected mathematical structure allows for rigorously defined modeling processes (Herrmann, 1991a). This mathematical structure has a great deal to say about the mathematical schism mentioned above. For example, instantaneous velocity, acceleration and even Newton's second law of motion can be derived from fundamental properties of the infinitesimal world rather than simply being defined (Herrmann, 1991a, part 2, pp. 4 –15). Moreover, infinitesimals do not behave in exactly the same manner as do directly observed real objects.

The schism mentioned above has been ignored by most modern day scientists. Such scientists simply use the old contradictory infinitesimal language to derive expressions that are claimed to model real world physical behavior without discussing the realism question for portions of their derivations. The mathematical model called *the nonstandard physical world model* (i.e. NSP-world) uses the corrected theory of the infinitesimally small and infinitely large, with other techniques, along with a new physical language theory of correspondence. Many of these new entities need not exist within the natural world since their behavior differs greatly from any known natural world entity.

The above schism in mathematical modeling has now become more evident for, from the viewpoint of *indirect evidence*, these new entities might actually exist in an NSP-world in which the natural world is specially embedded. Indeed, one could say that the NSP-world is omnipresent with respect to the natural world and upholds, sustains and guides natural system behavior. Notwithstanding this possibility, science cannot eliminate the NSP-world from its derivations if mathematical models are used. The NSP-world is always lurking in the background. Of recent interest is the possible existence of a natural world space medium (Barnes, 1986) that might be operationally obtained as the standard part of behavior within the NSP-world <u>nonstandard photon-particle medium</u> (i. e. NSPPM), where photon particle behavior is considered. Photon behavior uses a portion of the field of subparticles. This NSPPM is closely related to the basic Lorentz transformation. The NSPPM is not to be taken as a "field" that follows the nonstandard extension of the Maxwell field equations.

4. Relativity and Logical Error

Using the (S) philosophy, Einstein (1979) wrote that at the age of sixteen, using a *conceptual observer*, he had concocted a mind experiment relative to the known properties of light propagation that seemed to imply a paradox. He claimed to have eliminated this paradox some ten years later (Einstein, 1905). When this author was twelve, he read with interest Einstein's major book on this subject (Einstein, 1945) and felt that there might be some logical error in the basic derivations. However, a resolution of this error had to wait until the mathematics itself was correct by Robinson.

How did Einstein arrive at his derivation?

> Then I myself wanted to verify the flow of the æther with respect to the Earth, in other words, the motion of the Earth. When I first thought about this problem, I did not doubt the existence of the æther or the motion of the Earth through it (Einstein, 1982, p. 46).

Einstein did not state in his 1922 lecture that the æther did not exist. He said, "Since then I have come to believe that the motion of the Earth cannot be detected by any optical experiment though the Earth is revolving about the Sun" (1982, p. 46). However, later he and Infeld specifically argue against the æther in their popularizing book on scientific intuition (Einstein and Infeld, 1938, pp. 157–186). The argument is based entirely upon the (S) philosophy and erroneous hidden assumptions. The most glaring assumption is that if they could not describe an æther that satisfies the experimental conditions, then it does not exist.

The bases for Einstein's original 1905 paper is that "Time cannot be absolutely defined, and there is an inseparable relation between time and signal velocity (Einstein, 1982, p. 46)." Although Einstein states that an absolute time is not definable, it has been shown that this claim is false (Herrmann, 1992). This immediately implies that the Einstein derivation for the Lorentz transformation is inconsistent with respect to a basic premise. Einstein claims to be interested in an operational definition and first uses the term "clock" as meaning *any* measure of time within the natural world without further defining the apparatus. This does not immediately contradict the concept of absolute time as not being definable. But then he restricts the characterization of such clocks by adding light propagation terminology relative to their synchronization. Certain distances are also defined in terms of these restricted clocks and a property of light propagation. The predicate that is interpreted as "any time measure within the natural world" has been restricted to natural world clocks that are synchronizable by light propagation techniques. There always remains the possibility that not all identified natural world clocks are thus synchronizable. It can be argued that some biological clocks fall into this category.

Einstein next extends the domain of these "times" to include the local absolute Newtonian time continuum and applies the infinitesimal calculus to these"times." The infinitesimals represent a modeling *concept* used in the calculus to approximate a continuum of real numbers and there is no logical error if this technique is used consistently. The more closely the behavior of the measuring devices is modeled by real number properties, then the better this approximating mathematical device will predict behavior. As mentioned, Einstein introduced the "operational" time notion by requiring his "times" to be restricted by synchronization techniques and for "proper" time the "Einstein measure" technique. The basic logical error occurs later when these restrictions are dropped and the results are extended to "time" as a general concept. [For example, see Einstein 1907.] Such an error occurs when one substitutes nonequivalent predicates and is called the *model theoretic error of generalization*. A statement that holds for a specific domain (time restricted by the language of light propagation) cannot be extended to a domain that refers to time as measured by *any* device. For example, the statement that the usual ordering of the integers is a well-ordering does not hold when the domain is extended to the rational numbers.

Through use of a standard partial derivative technique, Einstein derives the Lorentz transformation. On the left-hand side of the equations is the proper time which is a measure of time defined in a slightly different manner using his synchronized natural world time concept. On the right-hand side of these equations, time is expressed in the original synchronized mode.

In the section of this 1905 paper entitled "Physical meaning of the equations obtained with respect to moving rigid bodies and moving clocks," the model theoretic error of generalization occurs. Einstein removes from his clock concepts the additional language of light propagation and generalizes the time concept to any measure of time whether correlated to light propagation or not. Throughout the remainder of this paper, the time measures, for any clock, utilizes the absolute Newtonian time continuum with infinitesimals so that the Calculus may be applied. Thus, in this paper, nonequivalent predicates are assumed equivalent.

Modern derivations of the Lorentz transformation (Bergmann, 1976; Lawden, 1982) do not apply Einstein's first infinitesimal approach but rather start with two coordinate statements $x^2 + y^2 + z^2 = c^2t^2$ and $\overline{x}^2 + \overline{y}^2 + \overline{z}^2 = c^2\overline{t}^2$. The time measures are once again restricted by light propagation language and the requirement of synchronization. Further, for this previous approach, these expressions are obtained by application of spherical wavefront (light) concepts, and the assumed constancy of the measure of the velocity of such propagation. Notice that these expressions require measurement of the propagation velocity to be made in accordance with devices that include the restricting light propagation language. After the derivation of the Lorentz transformation, these modern treatments once again employ the model theoretic error of generalization. First, claims are made that the results obtained hold not just for the light propagation associated measures of time and distance but for all natural world physical processes (Lawden, 1982, p. 13). Then the domain of application for the Lorentz transformation is further extended to a time continuum with infinitesimals (Lawden, 1982, p. 32). Not only do we have the same logical errors, but these modern treatments reject, without further thought, the æther concept as being a physical entity but use the absolute time concept which Einstein claimed also has no physical meaning.

Relative to the General Theory, the same logical error occurs. The square of the Minkowski space-time interval τ (Lawden, 1982, p. 14, eqt. (7.4)) for restricted time measures is assumed to hold for infinitesimals when τ is expressed in the famous differential form (Bergman, 1976, p. 44; Lawden, 1982, p. 132). Unless the unfounded extension of the restricted time measures to a time continuum with infinitesimals is used, the justification for this differential form in terms of "infinitesimal observers" who can synchronize their infinitesimal clocks (Lawden, 1982, p. 132) violates a basic requirement of infinitesimal modeling. This tenet states that to pass a time related property to the classical infinitesimal world it must hold with respect to a special approximation process for a Newtonian time interval that is modeled by a *continuum* of real numbers $\{x \mid a < x < b\}$. The use of any of the highly predictive forms of the classical infinitesimal calculus requires this assumption.

5. A Privileged Observer

Einstein's stated hypotheses, which contradicted all previous modeling assumptions, are described as:

I. The laws of nature are equally valid for all inertial frames of reference. II. The

> velocity of light is invariant for all inertial systems, being independent of the velocity of its source; more exactly, the measure of this velocity (of light) is a constant. (Prokhovnik, 1967, pp. 6–7).

These hypotheses, as well as the approach used for the General Theory, have significant philosophic implications. For these hypotheses reject

> . . . the Newtonian concept of a privileged observer, at rest in absolute space, . . . (Lawden, 1982, p. 127).

It is interesting how the appliers of the methods of mathematical modeling misunderstand even the most basic procedures. A privileged observer need not mean an actual entity within our universe. Whether or not one believes that a privileged observer exists in reality is a philosophic question. If a privileged observer is not a real entity, then it may be a member of the class of *conceptual* observers or an NSP-world observer. If, as Lawden claims, the privileged observer is to be rejected, then this cannot mean the rejection of the privileged conceptual observer or NSP-world observer since the entire intuitive foundations for the Special and General Theories of Relativity are based upon Einstein's mind experiment and concepts associated with a privileged conceptual observer or NSP-world observer.

The relation of these hypotheses to a Maxwellian type natural æther is that the æther is considered a place where privileged observation could take place. But actual "observation" using electromagnetic procedures by real human entities within our universe seems as if these observations can never be considered as taking place within such an æther for it appears that such entities cannot determine the basic property of their velocity through it. Hence, how would we know whether or not we are at rest with respect to such an æther? Further, unsatisfactory attempts were made to use perceived natural law to detail the behavior of such an æther assuming it is part of the natural world. Both of these apparent human inabilities contradict philosophy (S). Thus, to remove this contradiction, you simply postulate away the existence of real Maxwellian æther as a foundation. By this process, æther observers are removed if such an æther is within the universe. Since the theories use infinitesimal mathematics, you should not remove, in an ad hoc manner, NSP-world observation for the NSP-world contains a rigorously defined infinitesimal nonstandard substratum (the NS-substratum). Without such an NS-substratum, you incorporate into your science terms that are claimed to have no content, indeed, no meaning. Hence, the view is taken within this new research that privileged observation occurs within the pure NSP-world and that the fundamental frame of reference is an infinitesimal Cartesian frame with an Euclidean styled metric.

In practice, the idea of absolute length and time is used. Then, it is claimed, that such concepts have no meaning. We are told that *any* type of measurement of distance or time, no matter what kind, depends upon "relative velocity" without defining such a velocity. We are forbidden from searching for a "cause" for such behavior, a "cause" that may not be fully comprehensible. Special and General Theory behavior must simply be accepted without further thought.

For nonuniform velocity, the major mathematical structure that was available to Einstein and that upholds, almost completely, the philosophy of no privileged observer within the universe was the pure abstract mathematical structure known as *classical Riemannian geometry* (or the absolute or generalized calculus). It turns out that Einstein was not a good mathematician but he lived in a region of Germany which was a "hot bed" for studies of this structure. This mathematical structure was the only one available that appeared to match his intuition. He received considerable help that led to his guess as to a proper expression for a new law of gravity. If such a new approach was accepted, it would certainly enhance the importance of this mathematician's logical game.

Classical Riemannian geometry is defined entirely by the infinitesimal calculus in terms of a required transformation scheme between special types of coordinate systems. At least locally, each pair of coordinates must satisfy very special transformation rules. This means that the coordinate systems involved are not all of the possible ones. The use of classical tensor analysis actually contradicts the basic philosophy that "Physical space is, then, nothing more than the aggregate of all possible coordinate frames (Lawden, 1982, 127)." What can only be said is that the General Theory of Relativity, in its classical form, applies only to coordinate frames that are modeled by this structure, a model that, at that time, was thought to correspond correctly to infinitesimal intuition.

Conceptually, the geometry of surfaces requires "observation" from a higher dimension. But Einstein's General Theory uses well-known surface concepts where the surface has four dimensions. The statement that there can be no privileged observer within the universe does not preclude a corresponding type of conceptual observation from a required "outside" higher dimension. Although higher dimensions can be mathematically introduced into the theory, conceptual observation is usually based upon human experience. Notwithstanding the balloon analogy, it appears very difficult to give a truly meaningful nonmathematical description for the appearance of an assumed 4-dimensional "surface" within a 5-dimensional space.

After the development of our modern approaches to mathematical structures, it has become apparent that there is more than one mathematical structure that will lead to the same physical consequences. These other structures often model a different philosophy of science. For example, a fractal curve is supposed to be a highly nonsmooth entity. Yet, it has been shown (Herrmann, 1989) that such fractal effects can be produced by a highly smooth process within the NSP-world, an ultrasmooth microeffect. Thus two different philosophies for the physical theory of fractal curves can be utilized.

We now know that the concept of the infinitesimal as viewed physically prior to 1966 did not correspond to a mathematical structure. Further, in direct opposition to the modeling concepts of Newton, the mathematical structure often comes first and nature is required to behave as it dictates. Once again, we have philosophy (S).

The General Theory of Relativity associates pure geometric terms such as "intrinsic curvature" and the associated "geodesic curvature" for the "force" concept in Newton's theory and replaces action-at-a-distance with propagated gravitational effects. But General Relativity is a continuum theory. Hence, with a rejection of a space medium for radiation or gravitational propagation effects, there would be regions surrounding positions within our universe that over an actual interval of time are totally empty of known or imagined entities. Yet, the term intrinsic curvature would apply to these 4-dimensional regions. Further, the rules of the "game" again state that scientists should not be allowed to investigate a more basic "cause" for gravitational effects.

If other well established mathematical models predict the exact same consequences as the General Relativity, then substituting the nonintuitive concept of the intrinsic curvature of space-time for the experiential concept of force would be unnecessary. Is the concept of action-at-a-distance any less comprehensible to the human mind than an intrinsic curvature of "empty" 4-dimensional space-time?

6. The Fock Criticism and Significant Other Matters

In the 1930s, a major technical criticism of some of the Einsteinian arguments was brought forth by the Russian cosmologist Fock (1959). His criticism is related to the *Equivalence Principle* as used by Einstein. Consider, first, the global or overall space-time physical law of the equivalence of inertial mass (i.e. the "m" in the Newtonian expression $\vec{F} = m\vec{a}$) and gravitational mass (i.e. in Newtonian gravity theory, for example, the mass stated in this law). Then consider the so-called Equivalence Principle states that an acceleration field (i.e. the $\vec{a}$) and a gravitational field cannot be distinguished one from another. There have been many arguments presented that such an Equivalence Principle relative to the field concept is false from the global viewpoint.

> As was mentioned, Einstein considered that from the point of view of the Principle of Equivalence it is impossible to speak of absolute acceleration just as it is impossible to speak of absolute velocity. We consider this conclusion of Einstein's to be erroneous. . . . This conclusion is based upon the notion that fields of acceleration are indistinguishable from fields of gravitation. But, although the effects of acceleration and of gravitation may be indistinguishable "in the small", i.e. locally, they are undoubtedly distinguishable "in the large", i. e. when the boundary conditions to be imposed on gravitational field are taken into account (Fock, 1959, p. 208)

Fock gives an example of this for a rotating system and many others have given examples for accelerating noninfinitesimal structures. Then Fock writes:

> In the first place there is here an incorrect initial assumption. Einstein speaks of arbitrary gravitational fields extending as far as one pleases and not limited by boundary conditions. Such fields *cannot exist.* Boundary conditions or similar conditions which characterize space as a whole are absolutely essential and thus the notion of "acceleration relative to space" retains its significance in some form or another. . . . The essence of the error committed is in the initial assumption consists in *forgetting that the nature of the equivalence of fields of acceleration and of gravitation is strictly local* (1959, p. 369).

The fact that the effects must be local is why the infinitesimal calculus, in generalized form, is used as a means to model mathematically a general law of gravity. Further, Fock criticizes the concept of nonuniqueness relative to the general notion that any frame of reference will suffice. As I have pointed out the term *any* is not correct when the generalized calculus is used since the frame of reference, as modeled by a coordinate system, must have certain differential properties and be very special locally. Fock claims that the term *any* must be restricted greatly. This restriction is somewhat technical in character and refers to a claimed wave-like quality that certain solutions to the Einstein gravitational field equations seem to require. Fock claims that the correct coordinate system in which to discuss solutions is an "harmonic" system (Fock, 1959, p. 346-352). Further, it is claimed that solutions must have an additional boundary condition that at "infinity" they become the infinitesimal "chronotopic" line-element. [Note: this will be derived in article 3.] This is also called the Minkowski-type line-element (i.e. metric), but Fock calls it the Galilean line-element (metric). Sometimes it is termed the Euclidean requirement. Whatever terminology is employed, the Fock idea is that there are preferred coordinate systems in which measurements make sense.

> In the question of an isolated system of bodies the question of a coordinate system is answered in the same way as in the absence of a gravitational field: there exists a preferred system of coordinates (Galilean or harmonic) but it is also possible to use any other coordinate system. However, the geometric significance of the latter can only be established by comparing it with the preferred system (1959, p. 376).

I agree with Fock that application of the Equivalence Principle is only local. The necessity for harmonic coordinates has not been established on mathematical grounds except that it leads to unique and testable conclusions for our universe. But Fock did not have our present day knowledge for the correct rules for infinitesimal modeling and, hence, missed the point entirely. The Einstein gravitational equations as they are expressed in terms of the general or absolute calculus can be analogue modeled in many ways. *One* language that can be used is that of Riemannian geometry. But, Riemannian geometry is just that, an analogue model; something that simply represents behavior of something else but is not itself reality. This may lead to a certain mental visualization that aid in producing conclusions, but it also appears that visualizing the Einstein equations in terms of a generalization of the four dimensional wave equation also aids in such comprehension. This is precisely the reason that harmonic coordinates are introduced. Both of these realization techniques still remain analogue models for a reality that was not expressible in a correct language until now.

Recently, I have discovered that Fokker (1955) did guess at the correct concepts. Fokker suggested that the name *chronogeometry* would be more appropriate. As it will be established, Fokker was correct with this suggestion. When one realizes that mathematical coordinate systems are but an abstract entity without relation to reality, then one is lead to a theory of measurement that would correspond to a particular coordinate system. As will be shown in the articles that follow this one, the concept of the privileged observer comes prior to the selection of a coordinate system. After such an observer is defined, then a definable measuring process may be correlated to a compatible coordinate system. The privileged observer will essentially be observing infinitesimal light-clocks (defined in article 2) and nothing more.

For comparison purposes, the location, so to speak, of the fundamental observer is may be considered as fixed in the NSPPM. The NS-substratum coordinate system is a four dimensional Cartesian (Euclidean) system with the infinitesimal light-clocks being oriented along coordinate lines in order to measure the dynamic properties of motion. When certain motion occurs within the natural world, the infinitesimal light-clocks undergo an alteration in their internal structure in the sense that a newly constructed light-clcock is required. This alteration leads to a physical, not geometric, statement that can be considered an invariant under certain physical changes that are cause by motion of the infinitesimal light-clocks. This statement states that due to certain properties of electromagnetic radiation within the natural world, there will be a specific relation between coordinate (infinitesimal) light-clocks that is due to motion with respect to the NSPPM.

Prior to any physical alteration in the infinitesimal light-clock counting mechanism, an acceptable coordinate change (i.e. continuously differentiable with nonvanishing Jacobian) has the basic purpose of simply changing the "orientation," so to speak, of the infinitesimal light-clocks in order to give a different measure of the physical dynamics. On the other hand, using the General Relativity assumptions associated with Riemannian geometry and *once a solution is obtained for a particular physical scenario*, the coordinate change is interpreted as an acceptable alteration in the gravitational field. It is then claimed that such an alteration in the gravitational field will affect the abstract notion of "time." Such an interpretation is in error logically. Physical measures altered by a gravitational field are modeled by alterations in infinitesimal light-clock measures - a model that yields alterations in physical behavior.

The special physical "invariant" dS^2 is used. The invariant statement says that IF one imposes a coordinate change as a means of measuring different dynamic properties, then the expression dS^2 (i.e. the infinitesimal "chronotopic interval") remains fixed.

Fock and all previous researchers start their investigation with this so-called invariant statement, but, of course, never relate the statement to the actual entities that are being altered by motion, the light-clocks. The chronotopic interval can be generalized and represents light-clock behavior without the presence of a gravitational field of one sort or another as the agent for the motion in question. The major aspect of this interval is "uniform" velocity and, when infinitesimalized, nonuniform velocity. When generalized, the physical invariant (i.e. the fundamental metric of space-time, which I term a "line-element" since tensors are not used) looks like $dS^2 = g_{11}dx_1dx_1 + g_{12}dx_1dx_2 + \cdots + g_{44}dx_4dx_4$ and for such things as gravitations fields one can consider various types of "potential" velocities. However, the various dx_i cannot be uncontrolled infinitesimals in that they require an interpretation. This expression comes from classical Riemannian geometry and in that discipline each dx_i takes on different interpretations. In one case, each x_i is a function of another parameter (usually "time") and the expression is used to measure of arc-length along a "curve." In the general case, each dx_i is supposed to be interpreted as the geometric concept of "distance between infinitely near points." Prior to 1961 and Robinson's discovery, these interpretations did not follow a mathematically rigorous theory. Prior to the late 1980s, the actual method of obtaining an interpretation, the "infinitesimalizing method" also had not been discovered. The discovery of these correct methods shows that the actual infinitesimal chronotopic interval should not be interpreted in geometric terms, but it should be interpreted in terms of measures that retain an electromagnetic propagation language, and motion or potential motion. Of course, such differential models are, usually, intended to give approximations for macroscopic or large scale behavior. A correct method to incorporate both of these needed requirements will appear later.

For the basic infinitesimal chronotopic interval, the coefficients are as follows: the variable $dx_i,\ i = 1, 2, 3, 4$ are related to types of infinitesimal light-clock measurements. The coefficients are $g_{11} = g_{22} = g_{33} = -1,\ g_{44} = 1/c^2$, where c is the measured velocity of light *in vacuo.* All other coefficients are zero. Fock states, somewhat incorrectly, that to "understand" the geometry of space-time one needs to compare such changes in space-time geometry with this basic interval. In the presence of a gravitational field, the coefficients g_{ij} are related to the field's potential. This potential is also related to a force effect produced by the field, as previously mentioned, and, due to prior use of these concepts, leads to a geometric analogue model for what is, in reality, gravitational alterations in the behavior of electromagnetic radiation. If only the geometric method is used, then for a particular physical scenario, the terms of this line-element are appropriately altered so that the dx_idx_j represent specific geometric coordinate concepts and the corresponding functions g_{ij} satisfy the Einstein gravitational equation and also compensate for the dx_idx_j in such a manner that the line-element is invariant. In this manner and in terms of a geometric language, a comparison can be made to the line-element as it would be without the gravitational field. It will be argued in article 3, that dS^2 is actually a physical invariant only due to a special property associated with infinitesimal modeling.

The geometric approach actually contradicts the creator of the mathematics employed. The universe is NOT controlled by geometry. Geometry is a human construct that is used to model other intuitive concepts. Newton tells us that the intuitive concepts of mechanics come first. After experiencing behavior of forces, velocities, accelerations, resistance to motion, and the hundreds of other purely physical concepts, then the mathematical model is devised that will aid in logical argument and prediction. The mathematical model itself is not reality. The same holds for a Riemannian generalization of the geometry. Physical intuition should come first. Then certain rules for infinitesimal modeling are applied. The resulting constructions should aid in comprehending and predicting physical behavior as it is compared to the "simpler" and original physical intuition. The rules for such modeling are completely controlled by what is perceived of as simple behavior "in the small" that leads to complex behavior "in the large." We "understand" the more complex behavior by comparing it to the simpler behavior. Now Riemannian geometry can still be used, if it is properly interpreted in terms of the actual physical entities. But, if the foundation for your "gravitational forces" is stated in terms of a geometric language rather than in terms of intuitive physical qualities, then you would lose the great power of the infinitesimal calculus as a predictor of intuitive physical behavior.

For physical reasons, Patton and Wheeler (1975) also reject geometry as the ultimate foundations for our physical universe. Wheeler coined a new term "pregeometry" for the actual foundations.

> Riemannian geometry likewise provides a beautiful vision of reality; but it will be useful as anything we can do to see in what ways geometry is inadequate to serve as primordial building material. . . . "geometry" is as far from giving an understanding of space as "elasticity" is from giving an understanding of a solid. . . . (1975, p. 544, p. 557-558

In what follows, from the viewpoint of infinitesimal light-clocks, acceptable coordinate changes are first changes in the orientation of the measuring light clocks. After this is done, certain physical processes associated with the intuitive idea of a physical "potential velocity" are postulated. These are modeled by a simple linear correspondence. This leads to a general invariant line-element. As will be demonstrated in article 3, in order for this line-element to remain invariant, the differently oriented light-clocks would need to have one and only one aspect altered when substituted into this line-element. If one then associates the "potential velocities" with those that would be produced by a specific gravitational field, then solutions are obtained that are the same as those obtain from the Einstein equations and the Riemannian geometry approach, except that the solutions are stated in terms of actual physical processes that are being altered by these potentials. Other potential velocity substitutions lead to verified predictions for the Special Theory. The predictions obtained are in terms of infinitesimal light-clocks and how the altered light-clock behavior compares to that of the unaltered light-clock behavior.

7. Why Different Derivations?

There is a definite need for different derivations for theories that predict local events since all such theories are based upon philosophic foundations that impinge upon personal belief-systems. When the basic hypotheses for theory construction are identified, then these hypotheses always have a broader meaning called their *descriptive content.* The content of a collection of written statements, diagrams and other symbolic forms is defined as all of the mental impressions that the collection evokes within the mind of the reader. These impressions are, at the least, based upon an individual's experiences.

A personal belief-system also has content and this content can be contradicted by the content generated by the hypotheses or predictions of a scientific theory. Suppose that you have theories T_1 and T_2 each based upon different foundational concepts but the verified logical consequences of these two theories are the same. Further, the content of the foundations for T_1 does not contradict the content of your belief-system, while the content of the foundations for T_2 does. You now have two choices. You can accept theory T_1 as reasonable and reject T_2; or you can change your personal belief-system, accept theory T_2 and reject T_1. Whichever theory you select can be analyzed relative to its humanly comprehensible technical merits. Such an analysis does not necessarily imply that the selected theory is the correct theory.

There is absolute evidence that much of what passes for scientific theory is designed to force a rejection of various philosophic concepts, a rejection of well-founded belief-systems. For this reason, if for none other, it is important to identify the philosophic foundations of all scientific theories and to allow individuals a free-choice as to which they wish to accept.

With respect to both the Special and General Theories of Relativity, it is now possible to use the correct infinitesimal concepts, an NSPPM that does not reveal all of its properties, and obtain verified consequences of both theories without logical error. The error is eliminated by predicting alterations in clock behavior rather than by the error of inappropriate generalization. This new mathematical model alters the basic philosophic assumption of no privilege observer.

8. A Corrected Derivation

Notwithstanding Einstein's inability, using the philosophy (S), to describe a natural world æther that will lead verified Special Theory effects, there does exist a description that includes the necessary infinitesimals. A new derivation for the Lorentz transformation based upon absolute time with infinitesimals has been published (Herrmann, 1993). In this derivation, there is a *privileged observer using an inertial Cartesian coordinate system* as well as an additional, almost trivial, Galilean infinitesimal effect based upon natural world laboratory observations. The coordinate system lies in a portion of an NSP-world called the *nonstandard partical medium* – the NSPPM. This medium is a portion of the subparticle field and it yields relativistic alterations in natural-system behavior through an interaction with natural world entities associate with simple electromagnet propagation properties. The term *inertial* refers only to the weakest aspect of Galilean-Newtonian mechanics where, with respect to this NSPPM, a state of rest or uniform motion can be altered only by (force-like) interactions.

Few things can be known about the NSPPM. What is known is that certain basic expressions for Newtonian mechanics must be altered and even the general descriptions for such laws are slightly different. This NSPPM can be considered as part of the natural world if one wishes but it would be a very distinct part. The basic derivation is obtained using an absolute Newtonian time concept within NSPPM and a Galilean photon propagation theory that includes an infinitesimal statement which assumes that the velocity of light can be dependent upon an additional NSP-world velocity that could be that of the source. The apparent inability of measuring the velocity of the Earth through this medium using certain electromagnetic techniques and, hence, only being able to determine by such natural world techniques relative velocities is incorporated into this basic derivation. Further, the constancy of the to-and-fro natural world measurements of the velocity of light is included where, however, this velocity need not be constant with respect to the absolute time. Simply stated, this new derivation adjoins to certain electromagnetic behavior a simple additional infinitesimal property related to observed electromagnetic propagation.

The basic properties for NSPPM time as measured by a to-and-fro electromagnetic propagation experiment, once obtained from this derivation, are now applied to the natural world. In order to retain the electromagnetic character of the Special Theory effects, it is necessary that the Lorentz transformations be derived via hyperbolic geometry. This derivation method retains in its time and distance measures the electromagnetic propagation language. Such measures are termed as *Einstein measures* and are denoted by the subscript E. The Prokhovnik (1967) interpretation of the results relative to the Hubble textural expansion is, however, totally rejected since the ω is not related in any manner to such an assumed expansion. I point out that the use of the NSPPM eliminates not only the incomprehensible physical world length contraction and time dilation relations but even the difficulties associated with the reciprocals of these relations (i.e. the twin paradox).

A new refinement of the concept of Einstein measure is mentioned in this soon to be published 1993 paper. This refinement retains completely the electromagnetic propagation language by introducing the analogue model of the light-clock. The analogue light-clock model is composed of a fix rigid arm, of various lengths, that has a beginning light source attached to one end with a counter and returning mirror, and simply a returning mirror at the other end. The "counter" counts the number of to-and-fro paths an electromagnetic pulse "makes" from some fixed beginning count setting. Notice that from field properties an electromagnetic pulse's speed (at least one photon) is measured as constant. This concept is passed to the infinitesimal world where the arm's "length" can be considered a positive infinitesimal. What this yields is the *infinitesimal light-clock.* This does not preclude the possibility that under various conditions the speed of photons in the infinitesimal light-clocks is altered.

In the natural world, the light-clock concept can only approximate the continuum associated Einstein measures where the approximation is improved as the length of the arm is reduced. As will be discussed in article 2, this approximation may be made exact within the infinitesimal world. Identical infinitesimal light-clocks are used to measure the "time" by corresponding this concept to the counter number, the counter "ticks." Twice the arm's length multiplied by the counter number gives a measurement of an apparent distance, in terms of linear units, the electromagnetic pulse has traversed for a specific count number. Tracing the path of electromagnetic radiation leads to the basic interpretation that these light-clock counts can be used to measure, within the NSP-world, the to-and-fro electromagnetic path length within one moving light-clock. It is a measure in terms of linear units for nonlinear behavior.

Technically, in the NSP-world, twice the ruler-like measurement, in terms of private units, of the arm's length can be considered a positive infinitesimal L. It is known (Herrmann, 1991b, p. 108) that for every positive real number r there exists an "infinite" count number Π, where Π is a Robinson infinite number, such that r is *infinitesimally (or infinitely) close to* $L\Pi$. Notationally this is written as $r \approx L\Pi$, where $\approx$ is at the least an equivalence relation. What this means is that $r - L\Pi$ is an infinitesimal. There also exist infinite numbers Λ such that $L\Lambda$ is infinitesimally near to zero.

If $L\Pi$ is known to be infinitesimally near to real number, then $L\Pi$ is said to be *finite.* There is a process that can be used to capture the real number r when $L\Pi$ is known to be finite. This process uses the *standard part operator* that is denoted by "st." Many properties of the operator "st" are obtained from results in abstract algebra and these properties include the same formal properties as the "limit" viewed as an operator. The infinitesimal light-clocks yield an exact analogue for Einstein measures when the standard part operator is applied.

The discussion in Herrmann (1992) shows how the use of infinitesimal light-clocks allows for a return to the concept of absolute Newtonian time. Natural world observations lead to infinitesimal properties for the NSPPM expressed in terms of absolute time. Then, counter to the Einstein claim, these light-clock approximations are used to define infinitesimal light-clocks, which in turn lead to unique NSPPM times. Special light propagation properties lead to an Einstein time definition. But Einstein time can also be successively approximated within the natural world in terms of light-clock measures and only such measures. In terms of infinitesimal light-clocks, Einstein time can be exactly obtained. This discussion also shows that known Special Theory effects associated with uniform relative velocity (i.e. not incorporating possible gravitational effects) can be interpreted as manifestations of the electromagnetic character of natural world entities and how they interact with the NSPPM. What has not been investigated, is what relation gravity has with respect to the NSPPM and how such a gravity relation might influence the physical behavior within our natural world. In part 2 of this paper (Article 3), based upon a privileged observer located within the NSPPM, the infinitesimal chronotopic line-element is derived from light-clock properties and shown to be related to the propagation of electromagnetic radiation. A general expression is derived, without the tensor calculus, from basic infinitesimal theory applied to obvious Galilean measures for distances traversed by an electromagnetic pulse. Various line-elements are obtained from this general expression. These include the Schwarzschild (and modified) line-element, which is obtained by merely substituting a Newtonian gravitational velocity into this expression; the de Sitter and the Robertson-Walker which are obtained by substituting a velocity associated with the cosmological constant or an expansion (contraction) process. The relativistic (i.e. transverse Doppler), gravitational and cosmological redshifts, and alterations of the radioactive decay rate are derived from a general behavioral model associated with atomic systems, and it is predicted that similar types of shifts will take place for other specific cases. Further, the mass alteration expression is derived in a similar manner. From these derivations, locally verified predictions of the Einstein Special and General Theories of Relativity can be obtained. A process is also given that minimizes the problem of the “infinities” associated with such concepts as the Schwarzschild radius. These ideas are applied to the formation of black holes and pseudo-white holes.

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A Corrected Derivation for the Special Theory of Relativity*

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Abstract: Using properties of the nonstandard physical world, a new fundamental derivation for effects of the Special Theory of Relativity is given. This fundamental derivation removes all the contradictions and logical errors in the original derivation and leads to the fundamental expressions for the Special Theory Lorentz transformation. Necessarily, these are obtained by means of hyperbolic geometry. It is shown that the Special Theory effects are manifestations of the interaction between our natural world and a nonstandard medium, the NSPPM. This derivation eliminates the controversy associated with any physically unexplained absolute time dilation and length contraction. It is shown that there is no such thing as a absolute time dilation and length contraction but, rather, alterations in pure numerical quantities associated with an electromagnetic interaction with an NSP-world NSPPM.

1. The Fundamental Postulates.

There are various Principles of Relativity. The most general and least justified is the one as stated by Dingle "*There is no meaning in absolute motion.* By saying that such motion has *no meaning*, we assert that there is no observable effect by which we can determine whether an object is absolutely at rest or in motion, or whether it is moving with one velocity or another."[1:1] Then we have Einstein's statements that "I. The laws of motion are equally valid for all inertial frames of reference. II. The velocity of light is invariant for all inertial systems, being independent of the velocity of its source; more exactly, the measure of this velocity (of light) is constant, c, for all observers."[7:6–7] I point out that Einstein's original derivation in his 1905 paper (*Ann. der Phys.* **17**: 891) uses certain well-known processes related to partial differential calculus.

In 1981 [5] and 1991 [2], it was discovered that the intuitive concepts associated with the Newtonian laws of motion were inconsistent with respect to the mathematical theory of infinitesimals when applied to a theory for light propagation. The apparent nonballistic nature for light propagation when transferred to infinitesimal world would also yield a nonballistic behavior. Consequently, **there is an absolute contradiction between Einstein's postulate II and the derivation employed.** This contradiction would not have occurred if it had not been assumed that the æther followed the principles of Newtonian physics with respect to electromagnetic propagation. [Note: On Nov. 14, 1992, when the information in this article was formally presented, I listed various predicates that Einstein used and showed the specific places within the derivations where the predicate's domain was altered without any additional argument. Thus, I gave specific examples of the model theoretic error of generalization. See page 49.]

I mention that Lorentz speculated that æther theory need not correspond directly to the mathematical structure but could not show what the correct correspondence would be. Indeed, if one assumes that the NSPPM satisfies the most basic concept associated with an inertial system that a body can be considered in a state of rest or uniform motion unless acted upon by a force, then the expression $F = ma$, among others, may be altered for infinitesimal NS-substratum behavior. Further, the NS-substratum, when light propagation is discussed, does not follow the Galilean rules for velocity composition. The additive rules are followed but no negative real velocities exterior to the Euclidean monads are used since we are only interested in the propagation properties for

*This is an expanded version of the paper presented before the Mathematical Association of America, Nov. 14, 1992, Coppin State College, Baltimore Md electromagnetic radiation. The derivation in section 3 removes all contradictions by applying the most simplistic Galilean properties of motion, including the ballistic property, but only to behavior within a Euclidean monad.

As discussed in section 3, the use of an NSP-world (i.e. *nonstandard physical world*) NSPPM allows for the elimination of the well-known Special Theory "interpretation" contradiction that the mathematical model uses the concepts of Newtonian absolute time and space, and, yet, one of the major interpretations is that there is no such thing as absolute time or absolute space.

Certain general principles for NSPPM light propagation will be specifically stated in section 3. These principles can be gathered together as follows: (1) *There is a portion of the nonstandard photon-particle medium - the NSPPM - that sustains N-world (i.e. natural = physical world) electromagnetic propagation. Such propagation follows the infinitesimally presented laws of Galilean dynamics, when restricted to monadic clusters, and the monadic clusters follow an additive and an actual metric property for linear relative motion when considered collectively.* [The term "nonstandard electromagnetic field" should only be construed as a NSPPM notion, where the propagation of electromagnetic radiation follows slightly different principles than within the natural world.] (2) *The motion of light-clocks within the N-world (natural world) is associated with one single effect. This effect is an alteration in an appropriate light-clock mechanism.* [The light-clock concept will be explicitly defined at the end of section 3.] It will be shown later that an actual physical cause may be associated with vreified Special Theory physical alterations. Thus the Principle of Relativity, in its general form, and the inconsistent portions of the Einstein principles are eliminated from consideration and, as will be shown, the existence of a special type of medium can be assumed without contradicting experimental evidence.

In modern Special Theory interpretations [6], it is claimed that the effect of "length contraction" has no physical meaning, whereas time dilation does. This is probably true if, indeed, the Special Theory is actually based upon the intrinsic N-world concepts of length and time. What follows will further demonstrate that the Special Theory is a light propagation theory, as has been previously argued by others, and that the so-called "length contraction" and time dilation can both be interpreted as physically real effects when they are described in terms of the NSPPM. The effects are only relative to a theory of light propagation.

2. Pre-derivation Comments.

Recently [2]–[4], nonstandard analysis [8] has proved to be a very significant tool in investigating the mathematical foundations for various physical theories. In 1988 [4], we discussed how the methods of nonstandard analysis, when applied to the symbols that appear in statements from a physical theory, lead formally to a pregeometry and the entities termed as subparticles. One of the goals of NSP-world research is the re-examination of the foundations for various controversial N-world theories and the eventual elimination of such controversies by viewing such theories as but restrictions of more simplistic NSP-world concepts. This also leads to indirect evidence for the actual existence of the NSP-world.

The Special Theory of Relativity still remains a very controversial theory due to its philosophical implications. Prokhovnik [7] produced a derivation that yields all of the appropriate transformation formulas based upon a light propagation theory, but unnecessarily includes an interpretation of the so-called Hubble textural expansion of our universe as an additional ingredient. The new derivation we give in this article shows that properties of a NSPPM also lead to Prokhovnik's expression (6.3.2) in reference [7] and from which all of the appropriate equations can be derived. However, rather than considering the Hubble expansion as directly related to Special Relativity, it is shown that one only needs to consider simplistic NSP-world behavior for light propagation and the measurement of time by means of N-world light-clocks. This leads to the conclusion that Special Theory effects may be produced by a dense NSPPM within the NSP-world. Such an NSPPM − an æther − yields N-world Special Theory effects.

3. The derivation

The major natural system in which we exist locally is a space-time system. "Empty" space-time has only a few characterizations when viewed from an Euclidean perspective. We investigate, from the NSP-world viewpoint, electromagnetic propagation through a Euclidean neighborhood of space-time. Further, we assume that light is such a propagation. One of the basic precepts of infinitesimal modeling is the experimentally verified *simplicity* for such a local system. For actual time intervals, certain physical processes take on simplistic descriptions. These NSP-world descriptions are represented by the exact same description restricted to infinitesimal intervals. Let $[a, b],\ a \neq b,\ a > 0$, be an objectively real conceptional time interval and let $t \in (a, b)$.

The term "time" as used above is very misunderstood. There are various viewpoints relative to its use within mathematics. Often, it is but a term used in mathematical modeling, especially within the calculus. It is a catalyst so to speak. It is a modeling technique used due to the necessity for infinitesimalizing physical measures. The idealized concept for the "smoothed out" model for distance measure appears acceptable. Such an acceptance comes from the use of the calculus in such areas as quantum electrodynamics where it has great predictive power. In the subatomic region, the assumption that geometric measures have physical meaning, even without the ability to measure by external means, is justified as an appropriate modeling technique. Mathematical procedures applied to regions "smaller than" those dictated by the uncertainty principle are accepted although the reality of the infinitesimals themselves need not be assumed. On the other hand, for this modeling technique to be applied, the rules for ideal infinitesimalizing should be followed.

The infinitesimalizing of ideal geometric measures is allowed. But, with respect to the time concept this is not the case. Defining measurements of time as represented by the measurements of some physical periodic process is not the definition upon which the calculus is built. Indeed, such processes cannot be infinitesimalized. To infinitesimalize a physical measurement using physical entities, the entities being observed must be capable of being smoothed out in an ideal sense. This means that only the macroscopic is considered, the atomic or microscopic is ignored. Under this condition, you must be able to subdivide the device into "smaller and smaller" pieces. The behavior of these pieces can then be transferred to the world of the infinitesimals. Newton based the calculus not upon geometric abstractions but upon observable mechanical behavior. It was this mechanical behavior that Newton used to define physical quantities that could be infinitesimalized. This includes the definition of "time."

All of Newton's ideas are based upon velocities as the defining concept. The notation that uniform (constant) velocity exists for an object when that object is not affected by anything, is the foundation for his mechanical observations. This is an ideal velocity, a universal velocity concept. The modern approach would be to add the term "measured" to this mechanical concept. This will not change the concept, but it will make it more relative to natural world processes and a required theory of measure. This velocity concept is coupled with a smoothed out scale, a ruler, for measurement of distance. Such a ruler can be infinitesimalized. From observation, Newton then infinitesimalized his uniform velocity concept. This produces the theory of fluxions.

Where does observer time come into this picture? It is simply a defined quantity based upon the length and velocity concept. Observationally, it is the "thing" we call time that has passed when a test particle with uniform velocity first crosses a point marked on a scale and then crosses a second point marked on the same scale. This is in the absence of any physical process that will alter either the constant velocity or the scale. Again this definition would need to be refined by inserting the word "measured." Absolute time is the concept that is being measured and cannot be altered as aconcept.

Now with Einstein relativity, we are told that measured quantities are effected by various physical processes. All theories must be operational in that the concept of measure must be included. But, the calculus is used. Indeed, used by Einstein in his original derivation. Thus, unless there is an actual physical entity that can be substituted for the Newton's ideal velocity, then any infinitesimalizing process would contradict the actual rules of application of the calculus to the most basic of physical measures. But, the calculus is used to calculate the measured quantities. Hence, we are in a quandary. Either there is no physical basis for mathematical models based upon the calculus, and hence only selected portions can be realized while other selected portions are simply parameters not related to reality in any manner, or the calculus is the incorrect mathematical structure for the calculations. Fortunately, nature has provided us with the answer as to why the calculus, when properly interpreted, remains such a powerful tool to calculate the measures that describe observed physical behavior.

In the 1930s, it was realized that the measured uniform velocity of the to-and-fro velocity of electromagnetic radiation, (i.e. light) is the only known natural entity that will satisfy the Newtonian requirements for an ideal velocity and the concepts of space-time and from which the concept of time itself can be defined. The first to utilize this in relativity theory was Milne. This fact I learned after the first draughts of this paper were written and gives historical verification of this paper's conclusions. Although, it might be assumed that such a uniform velocity concept as the velocity of light or light paths *in vacuo* cannot be infinitesimalized, this is not the case. Such infinitesimalizing occurs for light-clocks and from the simple process of "scale changing" for a smoothed out ruler. What this means is that, at its most basic *physical* level, *conceptually* absolute or universal Newton time can have operational meaning as a physical foundation for a restricted form of "time" that can be used within the calculus.

As H. Dingle states it, "The second point is that the conformability of light to Newton mechanics . . . makes it possible to define corresponding units of space and time in terms of light instead of Newton's hypothetical 'uniformly moving body.' " [The Relativity of Time, *Nature*, 144(1939): 888–890.] It was Milne who first (1933) attempted, for the Special Theory, to use this definition for a "Kinematic Relativity" [*Kinematic Relativity*, Oxford University Press, Oxford, 1948] but failed to extend it successfully to the space-time environment. In what follows such an operational time concept is being used and infinitesimalized. It will be seen, however, that based upon this absolute time concept another time notion is defined, and this is the actual time notion that must be used to account for the physical changes that seem to occur due to relativistic processes. In practice, the absolute time is eliminated from the calculations and is replaced by defined "Einstein time." It is shown that Einstein time can be infinitesimalized through the use of the definable "infinitesimal light-clocks" and gives an exact measurement.

Our first assumption is based entirely upon the logic of infinitesimal analysis, reasoning, modeling and subparticle theory.

(i) "Empty" space within our universe, from the NSP-world viewpoint, is composed of a dense-like nonstandard medium (the NSPPM) that sustains, comprises and yields N-world Special Theory effects. These NSPPM effects are electromagnetic in character.

This medium through which the effects appear to propagate comprise the objects that yield these effects. The next assumption is convincingly obtained from a simple and literal translation of the concept of infinitesimal reasoning.

(ii) Any N-world position from or through which an electromagnetic effect appears to propagate, when viewed from the NSP-world, is embedded into a disjoint "monadic cluster" of the NSMP, where this monadic cluster mirrors the same unusual order properties, with respect to propagation, as the nonstandard ordering of the nonarchimedian field of hyperreal numbers ${}^*\mathbb{R}$. [2] A monadic cluster may be a set of NS-substratum subparticles located within a monad of the standard N-world position. The propagation properties within each such monad are identical.

In what follows, consider two (local) fundamental pairs of N-world positions $F_1,\ F_2$ that are in nonzero uniform (constant) NSP-world linear and relative motion. Our interest is in what effect such nonzero velocity might have upon such electromagnetic propagation. Within the NSP-world, this uniform and linear motion is measured by the number w that is near to a standard number ω and this velocity is measured with respect to conceptional NSP-world time and a stationary subparticle field. [Note that field expansion can be additionally incorporated.] The same NSP-world linear ruler is used in both the NSP-world and the N-world. The only difference is that the ruler is restricted to the N-world when such measurements are made. N-world time is measured by only one type of machine – the light-clock. The concept of the light-clock is to be considered as any clock-like apparatus that utilizes either directly or indirectly an equivalent process. As it will be detailed, due to the different propagation effects of electromagnetic radiation within the two "worlds," measured N-world light-clock time need not be the same as the NSP-world time. Further, the NSP-world ruler is the measure used to define the N-world light-clock.

Experiments show that for small time intervals $[a,b]$ the Galilean theory of average velocities (velocitys) suffices to give accurate information relative to the compositions of such velocities. Let there be an internal function $q\colon {}^*[a,b] \to {}^*\mathbb{R}$, where q represents in the NSP-world a distance function. Also, let nonnegative and internal $\ell\colon {}^*[a,b] \to {}^*\mathbb{R}$ be a function that yields the NSP-world velocity of the electromagnetic propagation at any $t \in {}^*[a,b]$. As usual $\mu(t)$ denotes the monad of standard t, where "t" is an absolute NSP-world "time" parameter.

The general and correct methods of infinitesimal modeling state that, within the internal portion of the NSP-world, two measures m_1 and m_2 are *indistinguishable for dt* (i.e. infinitely close of order one) (notation $m_1 \sim m_2$) if and only if $0 \neq dt \in \mu(0)$, ($\mu(0)$ the set of infinitesimals)

$$\frac{m_1}{dt} - \frac{m_2}{dt} \in \mu(0). \tag{3.1}$$

Intuitively, indistinguishable in this sense means that, although within the NSP-world the two measures are only equivalent and not necessarily equal, the *first level* (or first-order) effects these measures represent over dt are indistinguishable within the N-world (i.e. they appear to be equal.)

In the following discussion, we continue to use *photon* terminology. Within the N-world our photons need not be conceived of as particles in the sense that there is a nonzero finite N-world distance between individual photons. Our photons *may be* finite combinations of intermediate subparticles that exhibit, when the standard part operator is applied, basic electromagnetic field properties. They need not be discrete objects when viewed from the N-world, but rather they could just as well give the *appearance* of a dense NS-substratum. Of course, this dense NSPPM portion is not the usual notion of an "æther" (i.e. ether) for it is not a subset of the N-world. This dense-like portion of the NS-substratum contains*nonstandard particle medium* (NSPPM). Again "photon" can be considered as but a convenient term used to discuss electromagnetic propagation. Now for another of our simplistic physical assumptions.

(iii) In an N-world convex space neighborhood I traced out over the time interval $[a, b]$, the NSPPM disturbances appear to propagate linearly.

As we proceed through this derivation, other such assumptions will be identified.

The functions $q,\ \ell$ need to satisfy some simple mathematical characteristic. The best known within nonstandard analysis is the concept of S-continuity [8]. So, where defined, let $q(x)/x$ (a velocity type expression) and ℓ be S-continuous, and ℓ limited (i.e. finite) at each $p \in [a, b]$, ($a+$ at a, $b-$ at b). From compactness, $q(x)/x$ and ℓ are S-continuous, and ℓ is limited on ${}^*[a, b]$. Obviously, both q and ℓ may have infinitely many totally different NSP-world characteristics of which we could have no knowledge. But the function q represents within the NSP-world the distance traveled with linear units by an identifiable NSPPM disturbance. It follows from all of this that for each $t \in [a, b]$ and $t' \in \mu(t) \cap {}^*[a, b]$,

$$\frac{q(t')}{t'} - \frac{q(t)}{t} \in \mu(0);\ \ \ell(t') - \ell(t) \in \mu(0). \tag{3.2}$$

Expressions (3.2) give relations between nonstandard $t' \in \mu(t)$ and the standard t. Recall that if $x,\ y \in {}^*\mathbb{R}$, then $x \approx y$ iff $x - y \in \mu(0)$.

From (3.2), it follows that for each $dt \in \mu(0)$ such that $t + dt \in \mu(t) \cap {}^*[a, b]$

$$\frac{q(t + dt)}{t + dt} \approx \frac{q(t)}{t}, \tag{3.3}$$

$$\ell(t + dt) + \frac{q(t + dt)}{t + dt} \approx \ell(t) + \frac{q(t)}{t}. \tag{3.4}$$

One important observation is necessary. The fact that the function ℓ has been evaluated at $t + dt$ is not necessary for (3.4) to hold for it will also hold for any $t' \in \mu(t)$ and $\ell(t')$ substituted for $\ell(t + dt)$. But since we are free to choice any value $t' \in \mu(t)$, selecting particular values will allow our derivation to proceed to an appropriate N-world conclusion. From (3.4), we have that

$$\left(\ell(t + dt) + \frac{q(t + dt)}{t + dt}\right) dt \sim \left(\ell(t) + \frac{q(t)}{t}\right) dt. \tag{3.5}$$

It is now that we begin our application of the concepts of classical Galilean composition of velocities but restrict these ideas to the NSP-world monadic clusters and the notion of indistinguishable effects. You will notice that within the NSP-world the transfer of the classical concept of equality of constant or average quantities is replaced by the idea of indistinguishable. At the moment $t \in [a, b]$ that the standard part operator is applied, an effect is transmitted through the NSPPM as follows:

(iv) For each $dt \in \mu(0)$ and $t \in [a, b]$ such that $t + dt \in {}^*[a, b]$, the NSP-world distance $q(t+dt)-q(t)$ (relative to dt) traveled by the NSPPM effect within a monadic cluster is indistinguishable for dt from the distance produced by the Galilean composition of velocities.

From (iv), it follows that

$$q(t+dt)-q(t) \sim \left(\ell(t+dt)+\frac{q(t+dt)}{t+dt}\right)dt. \tag{3.6}$$

And from (3.5),

$$q(t+dt)-q(t) \sim \left(\ell(t)+\frac{q(t)}{t}\right)dt. \tag{3.7}$$

Expression (3.7) is the basic result that will lead to conclusions relative to the Special Theory of Relativity. In order to find out exactly what standard functions will satisfy (3.7), let arbitrary $t_1 \in [a,b]$ be the standard time at which electromagnetic propagation begins from position F_1. Next, let $q = {}^*s$ be an extended standard function and s is continuously differentiable on $[a,b]$. Applying the definition of $\sim$, yields

$$\frac{{}^*s(t+dt)-s(t)}{dt} \approx \ell(t)+\frac{s(t)}{t}. \tag{3.8}$$

Note that ℓ is microcontinuous on ${}^*[a,b]$. For each $t \in [a,b]$, the value $\ell(t)$ is limited. Hence, let $\mathtt{st}(\ell(t)) = v(t) \in \mathbb{R}$. From Theorem 1.1 in [3] or 7.6 in [10], v is continuous on $[a,b]$. [See note 1 part a.] Now (3.8) may be rewritten as

$${}^*\left(\frac{d(s(t)/t)}{dt}\right) = \frac{{}^*v(t)}{t}, \tag{3.9}$$

where all functions in (3.9) are *-continuous on ${}^*[a,b]$. Consequently, we may apply the *-integral to both sides of (3.9). [See note 1 part b.] Now (3.9) implies that for $t \in [a,b]$

$$\frac{s(t)}{t} = {}^*\!\!\int_{t_1}^{t} \frac{{}^*v(x)}{x}dx, \tag{3.10}$$

where, for $t_1 \in [a,b]$, $s(t_1)$ has been initialized to be zero.

Expression (3.10) is of interest in that it shows that although (iv) is a simplistic requirement for monadic clusters and the requirement that $q(x)/x$ be S-continuous is a customary property, they do not lead to a simplistic NSP-world function, even when view at standard NSP-world times. It also shows that the light-clock assumption was necessary in that the time represented by (3.10) is related to the distance traveled and unknown velocity of an identifiable NSPPM disturbance. It is also obvious that for pure NSP-world times the actual path of motion of such propagation effects is highly nonlinear in character, although within a monadic cluster the distance ${}^*s(t+dt)-s(t)$ is indistinguishable from that produced by the linear-like Galilean composition of velocities.

Further, it is the standard function in (3.10) that allows us to cross over to other monadic clusters. Thus, substituting into (3.7) yields, since the propagation behavior in all monadic clusters is identical,

$${}^*s(t+dt)-s(t) \sim \left({}^*v(t)+\left({}^*\!\textstyle\int_{t_1}^{t} {}^*v(x)/x dx\right)\right)dt, \tag{3.11}$$

for every $t \in [a,b],\ t+dt \in \mu(t) \cap {}^*[a,b]$

Consider a second standard position F_2 at which electromagnetic reflection occurs at $t_2 \in [a,b],\ t_2 > t_1,\ t_2+dt \in \mu(t_2) \cap {}^*[a,b]$. Then (3.11) becomes

$${}^*s(t_2+dt)-s(t_2) \sim \left({}^*v(t_2)+\left({}^*\!\textstyle\int_{t_1}^{t_2} {}^*v(x)/x dx\right)\right)dt. \tag{3.12}$$

Our final assumption for monadic cluster behavior is that the classical ballistic property holds with respect to electromagnetic propagation.

(v) From the exterior NSP-world viewpoint, at standard time $t \in [a, b]$, the velocity ${}^*v(t)$ acquires an additional velocity w.

Applying the classical statement (v), with the indistinguishable concept, means that the distance traveled ${}^*s(t_2 + dt) - s(t_2)$ is indistinguishable from $({}^*v(t_2) + w)dt$. Hence,

$$({}^*v(t_2) + w)dt \sim {}^*s(t_2 + dt) - s(t_2) \sim \left({}^*v(t_2) + \left({}^*\!\!\int_{t_1}^{t_2} \frac{{}^*v(x)}{x} dx \right) \right) dt. \tag{3.13}$$

Expression (3.13) implies that

$${}^*v(t_2) + w \approx {}^*v(t_2) + \left({}^*\!\!\int_{t_1}^{t_2} \frac{{}^*v(x)}{x} dx \right). \tag{3.14}$$

Since $\mathtt{st}(w)$ is a standard number, (3.14) becomes after taking the standard part operator,

$$\mathtt{st}(w) = \mathtt{st} \left({}^*\!\!\int_{t_1}^{t_2} \frac{{}^*v(x)}{x} dx \right). \tag{3.15}$$

After reflection, a NSPPM disturbance returns to the first position F_1 arriving at $t_3 \in [a, b]$, $t_1 < t_2 < t_3$. Notice that the function s does not appear in equation (3.15). Using the nonfavored position concept, a reciprocal argument entails that

$$\frac{s_1(t_3)}{t_3} = \mathtt{st} \left({}^*\!\!\int_{t_2}^{t_3} \frac{{}^*v_1(x)}{x} dx \right), \tag{3.16}$$

$$\mathtt{st}(w) = \mathtt{st} \left({}^*\!\!\int_{t_2}^{t_3} \frac{{}^*v_1(x)}{x} dx \right), \tag{3.17}$$

where $s_1(t_2)$ is initialized to be zero. It is not assumed that ${}^*v_1 = {}^*v$.

We now combine (3.10), (3.15), (3.16), (3.17) and obtain an interesting nonmonadic view of the relationship between distance traveled by an NSPPM disturbance and relative velocity.

$$s_1(t_3) - s(t_2) = \mathtt{st}(w)(t_3 - t_2). \tag{3.18}$$

Although reflection has been used to determine relation (3.18) and a linear-like interpretation involving reflection seems difficult to express, there is a simple nonreflection analogue model for this behavior.

Suppose that a NSPPM disturbance is transmitted from a position F_1, to a position F_2. Let F_1 and F_2 have no NSP-world relative motion. Suppose that a NSPPM disturbance is transmitted from F_1 to F_2 with a constant velocity v with the duration of the transmission $t'' - t'$, where the path of motion is considered as linear. The disturbance continues linearly after it passes point F_2 but has increased during its travel through the monadic cluster at F_2 to the velocity $v + \mathtt{st}(w)$. The disturbance then travels linearly for the same duration $t'' - t'$. The linear difference in the two distances traveled is $w(t'' - t')$. Such results in the NSP-world should be construed only as behavior mimicked by the analogue NSPPM model.

Equations (3.10) and (3.15) show that in the NSP-world NSPPM disturbances propagate. Except for the effects of material objects, it is assumed that in the N-world the path of motion displayed by a NSPPM disturbance is linear. This includes the path of motion within an N-world light-clock. We continue this derivation based upon what, at present, appears to be additional parameters, a private NSP-world time and an NSP-world rule. Of course, the idea of the N-world light-clock is being used as a fixed means of identifying the different effects the NSPPM is having upon these two distinct worlds. A question yet to be answered is how can we compensate for differences in these two time measurements, the NSP-world private time measurement of which we can have no knowledge and N-world light-clocks.

The weighted mean value theorem for integrals in nonstandard form, when applied to equations (3.15) and (3.17), states that there are two NSP-world times $t_a,\ t_b \in {}^*[a,b]$ such that $t_1 \leq t_a \leq t_2 \leq t_b \leq t_3$ and

$$\mathtt{st}(w) = \mathtt{st}({}^*v(t_a)) \int_{t_1}^{t_2} \frac{1}{x}dx = \mathtt{st}({}^*v_1(t_b)) \int_{t_2}^{t_3} \frac{1}{x}dx. \tag{3.19}$$

[See note 1 part c.] Now suppose that within the local N-world an $F_1 \to F_2,\ F_2 \to F_1$ light-clock styled measurement for the velocity of light using a fixed instrumentation yields equal quantities. (Why this is the case is established in Section 6.) Model this by (*) $\mathtt{st}({}^*v(t_a)) = \mathtt{st}({}^*v_1(t_b)) = c$ in the NSPPM. I point out that there are many nonconstant *-continuous functions that satisfy property (*). For example, certain standard nonconstant linear functions and nonlinear modifications of them. Property (*) yields

$$\int_{t_1}^{t_2} \frac{1}{x}dx = \int_{t_2}^{t_3} \frac{1}{x}dx. \tag{3.20}$$

And solving (3.20) yields

$$\ln\left(\frac{t_2}{t_1}\right) = \ln\left(\frac{t_3}{t_2}\right). \tag{3.21}$$

From this one has

$$t_2 = \sqrt{t_1 t_3}. \tag{3.22}$$

Expression (3.22) is Prokhovnik's equation (6.3.3) in reference [7]. However, (6.3.3) is based upon an ad hoc derivative assumption. Further, the interpretation of this result and the others that follow cannot, for the NSP-world, be those as proposed by Prokhovnik. The times $t_1,\ t_2,\ t_3$, are standard NSPPM times. Further, it is not logically acceptable when considering how to measure such time in the NSP-world or N-world to consider just any mode of measurement. The mode of light velocity measurement must be carried out within the confines of the language used to obtain this derivation. Using this language, a method for time calculation that is permissible in the N-world is the light-clock method. Any other described method for time calculation should not include significant terms from other sources. Time as expressed in this derivation is not a mystical *absolute* something or other. It is a measured quantity based entirely upon some mode of measurement.

They are two major difficulties with most derivations for expressions used in the Special Theory. One is the above mentioned absolute time concept. The other is the ad hoc nonderived N-world relative velocity. In this case, no consideration is given as to how such a relative velocity is to be measured so that from both F_1 and F_2 the same result would be obtained. It is possible to achieve such a measurement method because of the logical existence of the NSPPM.

In a physical-like sense, the "times" can be considered as the numerical values recorded by single device stationary in the NSPPM. It is conceptual time in that, when events occur, then such numerical event-times "exist." It is the not yet identified NSPPM properties that yield the unusual behavior indicated by (3.22). One can use light-clocks and a counter that indicates, from some starting count, the number of times the light pulse has traversed back and forth between the mirror and source of our light-clock. Suppose that F_1 and F_2 can coincide. When they do coincide, the F_2 light-clock counter number that appears conceptually first after that moment can be considered to coincide with the counter number for the F_1 light-clock.

After F_2 is perceived to no longer coincide with F_1, a light pulse is transmitted from F_1 towards F_2 in an assumed linear manner. The "next" F_1 counter number after this event is τ_{11}. We assume that the relative velocity of F_2 with respect to F_1 may have altered the light-clock counter numbers, compared to the count at F_1, for a light-clock riding with F_2. The length L used to define a light-clock is measured by the NSP-world ruler and would not be altered. Maybe the light velocity c, as produced by the standard part operator, is altered by N-world relative velocity. Further, these two N-world light-clocks are only located at the two positions $F_1,\ F_2$, and this light pulse is represented by a NSPPM disturbance. The light pulse is reflected back to F_1 by a mirror similar to the light-clock itself. The first counter number on the F_2 light-clock to appear, intuitively, "after" this reflection is approximated by τ_{21}. The F_1 counter number first perceived after the arrival of the returning light pulse is τ_{31}.

From a linear viewpoint, at the moment of reflection, denoted by τ_{21}, the pulse has traveled an operational linear light-clock distance of $(\tau_{21} - \tau_{11})L$. After reflection, under our assumptions and nonfavored position concept, a NSPPM disturbance would trace out the same operational linear light-clock distance measured by $(\tau_{31} - \tau_{21})L$. Thus the operational light-clock distance from F_1 to F_2 would be at the moment of operational reflection, under our linear assumptions, 1/2 the sum of these two distances or $S_1 = (1/2)(\tau_{31} - \tau_{11})L$. Now we can also determine the appropriate operational relation between these light-clock counter numbers for $S_1 = (\tau_{21} - \tau_{11})L$. Hence, $\tau_{31} = 2\tau_{21} - \tau_{11}$, and τ_{21} operationally behaves like an Einstein measure.

After, measured by light-clock counts, the pulse has been received back to F_1, a second light pulse (denoted by a second subscript of 2) is immediately sent to F_2. Although $\tau_{31} \leq \tau_{12}$, it is assumed that $\tau_{31} = \tau_{12}$ [See note 2.5]. The same analysis with new light-clock count numbers yields a different operational distance $S_2 = (1/2)(\tau_{32} - \tau_{12})L$ and $\tau_{32} = 2\tau_{22} - \tau_{12}$. One can determine the operational light-clock time intervals by considering $\tau_{22} - \tau_{21} = (1/2)((\tau_{32} - \tau_{31}) + (\tau_{12} - \tau_{11}))$ and the operational linear light-clock distance difference $S_2 - S_1 = (1/2)((\tau_{32} - \tau_{31}) - (\tau_{12} - \tau_{11}))L$. Since we can only actually measure numerical quantities as discrete or terminating numbers, it would be empirically sound to write the N-world time intervals for these scenarios as $t_1 = \tau_{12} - \tau_{11}, t_3 = (\tau_{32} - \tau_{31})$. This yields the operational Einstein measure expressions in (6.3.4) of [7] as $\tau_{22} - \tau_{21} = t_E$ and operational light-length $r_E = S_2 - S_1$, using our specific light-clock approach. This allows us to define, operationally, the N-world relative velocity as $v_E = r_E/t_E$. [In this section, the $t_1,\ t_3$ are not the same Einstein measures, in form, as described in [7]. But, in section 4, 5, 6 these operational measures are used along with infinitesimal light-clock counts to obtain the exact Einstein measure forms for the time measure. This is: the t_1 is a specific starting count and the t_3 is t_1 plus an appropriate lapsed time.]

Can we theoretically turn the above approximate operational approach for discrete N-world light-clock time into a time continuum? Light-clocks can be considered from the NSP-world viewpoint. In such a case, the actual NSP-world length used to form the light-clock might be considered as a nonzero infinitesimal. Thus, at least, the numbers $\tau_{32},\ \tau_{21},\ \tau_{31},\ \tau_{22}$ are infinite hyperreal numbers, various differences would be finite and, after taking the standard part operator, all of the N-world times and lengths such as $t_E,\ r_E,\ S_1,\ S_2$ should be exact and not approximate in character.

These concepts will be fully analyzed in section 6. Indeed, as previously indicated, for all of this to hold the velocity c cannot be measured by any means. As indicated in section 6, the actual numerical quantity c as it appears in (3.22) is the standard part of pure NSP-world quantities. Within the N-world, one obtains an "apparent" constancy for the velocity of light since, for this derivation, it must be measured by means of a to-and-fro light-clock styled procedure with a fixed instrumentation.

As yet, we have not discussed relations between N-world light-clock measurements and N-world physical laws. It should be self-evident that the assumed linearity of the light paths in the N-world can be modeled by the concept of projective geometry. Relative to the paths of motion of a light path in the NSP-world, the NSPPM disturbances, the N-world path behaves as if it were a projection upon a plane. Prokhovnik analyzes such projective behavior and comes to the conclusions that in two or more dimensions the N-world light paths would follow the rules of hyperbolic geometry. In Prokhovnik, the equations (3.22) and the statements establishing the relations between the operational or exact Einstein measures $t_E,\ r_E$ and v_E lead to the Einstein expression relating the light-clock determined relative velocities for three linear positions having three NSP-world relative and uniform velocities $w_1,\ w_2,\ w_3$.

In the appendix, in terms of light-clock determined Einstein measures and based upon the projection idea, the basic Special Theory coordinate transformation is correctly obtained. Thus, all of the NSP-world times have been removed from the results and even the propagation differences with respect to light-clock measurements. Just use light-clocks in the N-world to measure all these quantities in the required manner and the entire Special Theory is forthcoming.

I mention that it can be shown that w and c may be measured by probes that are not N-world electromagnetic in character. Thus w need not be obtained in the same manner as is v_E except that N-world light-clocks would be used for N-world time measurements. For this reason, $\mathtt{st}(w) = \omega$ is not directly related to the so-called textual expansion of the space within our universe. The NSPPM is not to be taken as a nonstandard translation of the Maxwell EMF equations.

4. The Time Continuum.

With respect to models that use the classical continuum approach (i.e. variables are assumed to vary over such things as an interval of real numbers) does the mathematics perfectly measure quantities within nature – quantities that cannot be perfectly measured by a human being? Or is the mathematics only approximate in some sense? Many would believe that if "nature" is no better than the human being, then classical mathematics is incorrect as a perfect measure of natural system behavior. However, this is often contradicted in the limit. That is when individuals refine their measurements, as best as it can done at the present epoch, then the discrete human measurements seem to approach the classical as a limit. Continued exploration of this question is a philosophical problem that will not be discussed in this paper, but it is interesting to model those finite things that can, apparently, be accomplished by the human being, transfer these processes to the NSP-world and see what happens. For what follows, when the term "finite" (i.e. limited) hyperreal number is used, since it is usually near to a nonzero real number, it will usually refer to the ordinary nonstandard notion of finite except that the infinitesimals have been removed. This allows for the existence of finite multiplicative inverses.

First, suppose that $t_E = \mathtt{st}(t_{Ea}),\ r_E = \mathtt{st}(r_{Ea}),\ S_1 = \mathtt{st}(S_{1a}),\ S_2 = \mathtt{st}(S_{2a})$ and each is a nonnegative real number. Thus $t_{Ea},\ r_{Ea},\ S_{1a},\ S_{2a}$ are all nonnegative finite hyperreal numbers. Let $L = 1/10^{\omega} > 0, \omega \in \mathbb{N}^{+}_{\infty}$. By transfer and the result that $S_{1a},\ S_{2a}$, are considered finite (i.e.

near standard), then $S_{1a} \approx (1/2)L(\tau_{31} - \tau_{11}) \approx L(\tau_{21} - \tau_{11}) \Rightarrow (1/2)(\tau_{31} - \tau_{11}),\ (\tau_{21} - \tau_{11})$ cannot be finite. Thus, by Theorem 11.1.1 [9], it can be assumed that there exist $\eta, \gamma \in \mathbb{N}_\infty^+$ such that $(1/2)(\tau_{31} - \tau_{11}) = \eta,\ (\tau_{21} - \tau_{11}) = \gamma$. This implies that each τ corresponds to an infinite light-clock count and that

$$\tau_{31} = 2\eta + \tau_{11},\ \tau_{21} = \gamma + \tau_{11}. \tag{4.1}$$

In like manner, it follows that

$$\tau_{32} = 2\lambda + \tau_{12},\ \tau_{22} = \delta + \tau_{12},\ \lambda, \delta \in \mathbb{N}_\infty^+. \tag{4.2}$$

Observe that the second of the double subscripts being 2 indicates the light-clock counts for the second light transmission.

Now for t_{Ea} to be finite requires that the corresponding nonnegative $t_{1a},\ t_{3a}$ be finite. Since a different mode of conceptual time might be used in the NSP-world, then there is a need for a number $u = L/c$ that adjusts NSP-world conceptual time to the light-clock count numbers. [See note 18.] By transfer of the case where these are real number counts, this yields that $t_{3a} \approx u(\tau_{32} - \tau_{31}) = 2u(\lambda - \eta) + u(\tau_{12} - \tau_{11}) \approx 2u(\lambda - \eta) + t_{1a}$ and $t_{Ea} \approx u(\tau_{22} - \tau_{21}) \approx u(\delta - \gamma) + t_{1a}$. Hence for all of this to hold in the NSP-world $u(\delta - \gamma)$ must be finite or that there exists some $r \in \mathbb{R}^+$ such that $u(\delta - \gamma) \in \mu(r)$. Let $\tau_{12} = \alpha,\ \tau_{11} = \beta$. Then $t_{Ea} \approx u(\delta - \gamma) + u(\alpha - \beta)$ implies that $u(\alpha - \beta)$ is also finite.

The requirement that these infinite numbers exist in such a manner that the standard part of their products with L [resp. u] exists and satisfies the continuum requirements of classical mathematics is satisfied by Theorem 11.1.1 [9], where in that theorem $10^\omega = 1/L$ [resp. $1/u$]. [See note 2.] It is obvious that the nonnegative numbers needed to satisfy this theorem are nonnegative infinite numbers since the results are to be nonnegative and finite. Theorem 11.1.1 [9] allows for the appropriate $\lambda,\ \eta,\ \delta,\ \gamma$ to satisfy a bounding property in that we know two such numbers exist such that $\lambda,\ \eta < 1/L^2,\ \delta,\ \gamma < 1/u^2$. [Note: It is important to realize that due to this correspondence to a continuum of real numbers that the entire analysis as it appears in section 3 is now consistent with a mode of measurement. Also the time concept is replaced in this analysis with a "count" concept. This count concept will be interpreted in section 8 as a count per some unit of time measure.]

Also note that the concepts are somewhat simplified if it is assumed that $\tau_{12} = \tau_{31}$. In this case, substitution into 4.1 yields that $t_{1a} \approx 2u\eta$ and $t_{3a} \approx 2u\lambda$. Consequently, $t_{Ea} = (1/2)(t_{1a} + t_{3a}) \approx u(\lambda+\eta)$. This predicts what is to be expected, that, in this case, the value of t_E from the NSP-world viewpoint is not related to the first "synchronizing" light pulse sent.

5. Standard Light-clocks and c.

I mention that the use of subparticles or the concept of the NSPPM are not necessary for the derivation in section 3 to hold. One can substitute for the NSPPM the term "NS-substratum" or the like and for the term "monadic cluster" of possible subparticles just the concept of a "monadic neighborhood." It is not necessary that one assume that the NS-substratum is composed of subparticles or any identifiable entity, only that NSPPM transmission of such radiation behaves in the simplistic manner stated.

It is illustrative to show by a diagram of simple light-clock counts how this analysis actually demonstrates the two different modes of propagation, the NSP-world mode and the different mode when viewed from the N-world. In general, L is always fixed and for the following analysis and, for this particular scenario, inf. light-clock c may change. This process of using N-world light-clocks to approximate the relative velocity should only be done once due to the necessity of "indexing" the light-clocks when F_1 and F_2 coincide. In the following diagram, the numbers represent actual light-clock count numbers as perceived in the N-world. The first column are those recorded at F_1, the second column those required at F_2. The arrows and the numbers above them represent our F_1 comprehension of what happens when the transmission is considered to take place in the N-world. The Einstein measures are only for the F_1 position.

F_1	N − world	F_2
$\tau_{11} = 20$		
	20 $\searrow$	
$\tau = 40$		$\tau_{21} = 40$
	20 $\swarrow$	
$\tau_{31} = 60$		
$\tau_{12} = 80$		
	30 $\searrow$	
		$\tau_{22} = 110$
	30 $\swarrow$	
$\tau_{32} = 140$		

Certainly, the above diagram satisfies the required light-clock count equations. The only light-clock counts that actually are perceivable are those at F_1. And, for the transformation equations, the scenario is altered. When the Special Theory transformation equations are obtained, two distinct N-world observers are used and a third N-world distinct fundamental position. All light-clock counts made at each of these three positions are entered into the appropriate expressions for the Einstein measures **as obtained for each individual position.**

6. Infinitesimal Light-clock Analysis.

In the originally presented Einstein derivation, time and length are taken as absolute time and length. It was previously pointed out that this assumpt yields logical error. The scientific community extrapolated the language used in the derivation, a language stated only in terms of light propagation behavior, without logical reason, to the "concept" of Newtonian absolute time and length. Can the actual meaning of the "time" and "length" expressed in the Lorentz transformation be determined?

In what follows, a measure by light-clock counts is used to analyze the classical transformation as derived in the Appendix-A and, essentially, such "counts" will replace conceptional time. [See note 1.5] The superscripts indicate the counts associated with the light-clocks, the Einstein measures, and the like, at the positions $F_1,\ F_2$. The 1 being the light-clock measures at F_1 for a light pulse event from P, the 2 for the light-clock measures at the F_2 for the same light pulse event from P, and the 3 for the light-clock measures and its corresponding Einstein measures at F_1 for the velocity of F_2 relative to F_1. The NSP-world measured angle, assuming linear projection due to the constancy of the velocities, from F_1 to the light pulse event from P is θ, and that from F_2 to P is an exterior angle ϕ.

The expressions for our proposes are $x_E^{(1)} = v_E^{(1)} t_E^{(1)} \cos\theta,\ x_E^{(2)} = -v_E^{(2)} t_E^{(2)} \cos\phi$. [Note: The negative is required since $\pi/2 \leq \phi \leq \pi$ and use of the customary coordinate systems.] In all that follows, i varies from 1 to 3. We investigate what happens when the standard model is now embedded back again into the *non-infinitesimal finite* NSP-world. All of the "coordinate" transformation equations are in the Appendix and they actually only involve ω_i/c. These equations are interpreted in the NSP-world. But as far as the light-clock counts are concerned, their appropriate differences are only infinitely near to a standard number. The appropriate expressions are altered to take this into account. For simplicity in notation, it is again assumed that "immediate" in the light-clock count process means $\tau^{(i)}_{12} = \tau^{(i)}_{31}$. [See note 3.] Consequently, $t^{(i)}_{1a} \approx 2u\eta^{(i)},\ t^{(i)}_{3a} \approx 2u\lambda^{(i)},\ \eta^{(i)}, \lambda^{(i)} \in \mathbb{N}^+_\infty$. Then

$$t^{(i)}_{Ea} \approx u(\lambda^{(i)} + \eta^{(i)}),\ \lambda^{(i)}, \eta^{(i)} \in \mathbb{N}^+_\infty. \tag{6.1}$$

Now from our definition $r^{(i)}_E \approx L(\lambda^{(i)} - \eta^{(i)}),\ (\lambda^{(i)} - \eta^{(i)}) \in \mathbb{N}^+_\infty$. Hence, since all of the numbers to which $\mathtt{st}$ is applied are nonnegative and finite and $\mathtt{st}(v^{(i)}_{Ea})\ \mathtt{st}(t^{(i)}_{Ea}) = \mathtt{st}(r^{(i)}_{Ea})$, it follows that

$$v^{(i)}_{Ea} \approx L\frac{(\lambda^{(i)} - \eta^{(i)})}{u(\lambda^{(i)} + \eta^{(i)})}. \tag{6.2}$$

Now consider a set of two 4-tuples

$$(\mathtt{st}(x^{(1)}_{Ea}), \mathtt{st}(y^{(1)}_{Ea}), \mathtt{st}(z^{(1)}_{Ea}), \mathtt{st}(t^{(1}_{Ea})),$$

$$(\mathtt{st}(x^{(2)}_{Ea}), \mathtt{st}(y^{(2)}_{Ea}), \mathtt{st}(z^{(2)}_{Ea}), \mathtt{st}(t^{(2)}_{Ea})),$$

where they are viewed as Cartesian coordinates in the NSP-world. First, we have $\mathtt{st}(x^{(1)}_{Ea}) = \mathtt{st}(v^{(1)}_{Ea})\mathtt{st}((t^{(1)}_{Ea})\mathtt{st}({}^*\!\cos\theta),\ \mathtt{st}(x^{(2)}_{Ea}) = \mathtt{st}(v^{(2)}_{Ea})\mathtt{st}(t^{(2)}_{Ea})\mathtt{st}({}^*\!\cos\phi)$. Now suppose the local constancy of c. The N-world Lorentz transformation expressions are

$$\mathtt{st}(t^{(1)}_{Ea}) = \beta_3(\mathtt{st}(t^{(2)}_{Ea}) + \mathtt{st}(v^{(3)}_{Ea})\mathtt{st}(x^{(2)}_{Ea})/c^2),$$

$$\mathtt{st}(x^{(1)}_{Ea}) = \beta_3(\mathtt{st}(x^{(2)}_{Ea}) + \mathtt{st}(v^{(3)}_{Ea})\mathtt{st}(t^{(2)}_{Ea})),$$

where $\beta_3 = \mathtt{st}((1 - (v^{(3)}_{Ea})^2/c^2)^{-1/2})$. Since $L(\lambda^{(i)} - \eta^{(i)}) \approx cu(\lambda^{(i)} - \eta^{(i)})$, the finite character of $L(\lambda^{(i)} - \eta^{(i)}),\ u(\lambda^{(i)} - \eta^{(i)})$ yields that $c = \mathtt{st}(L/u)$ [See note 8]. When transferred to the NSP-world with light-clock counts, substitution yields

$$t^{(1)}_{Ea} \approx u(\lambda^{(1)} + \eta^{(1)}) \approx \beta[u(\lambda^{(2)} + \eta^{(2)}) - u(\lambda^{(2)} + \eta^{(2)})K^{(3)}K^{(2)}\,{}^*\!\cos\phi], \tag{6.3}$$

where $K^{(i)} = (\lambda^{(i)} - \eta^{(i)})/(\lambda^{(i)} + \eta^{(i)}),\ \beta = (1 - (K^{(3)})^2)^{-1/2}$.

For the "distance" transformation, we have

$$x^{(1)}_{Ea} \approx L(\lambda^{(1)} - \eta^{(1)})\,{}^*\!\cos\theta \approx$$

$$\beta(-L(\lambda^{(2)} - \eta^{(2)})\,{}^*\!\cos\phi + \frac{L(\lambda^{(3)} - \eta^{(3)})}{u(\lambda^{(3)} + \eta^{(3)})}u(\lambda^{(2)} + \eta^{(2)})). \tag{6.4}$$

Assume in the NSP-world that $\theta \approx \pi/2,\ \phi \approx \pi$. Consequently, substituting into 6.4 yields

$$-L(\lambda^{(2)} - \eta^{(2)}) \approx \frac{L(\lambda^{(3)} - \eta^{(3)})}{u(\lambda^{(3)} + \eta^{(3)})}u(\lambda^{(2)} + \eta^{(2)}). \tag{6.5}$$

Applying the finite property for these numbers, and, for this scenario, taking into account the different modes of the corresponding light-clock measures, yields

$$\frac{L(\lambda^{(3)} - \eta^{(3)})}{u(\lambda^{(3)} + \eta^{(3)})} \approx \frac{-L(\eta^{(2)} - \lambda^{(2)})}{u(\lambda^{(2)} + \eta^{(2)})} \Rightarrow v^{(3)}_{Ea} \approx -v^{(2)}_{Ea}. \tag{6.6}$$

Hence, $\mathtt{st}(v_{Ea}^{(3)}) = -\mathtt{st}(v_{Ea}^{(2)})$. [Due to the coordinate-system selected, these are directed velocities.] This predicts that, in the N-world, the light-clock determined relative velocity of F_2 as measured from the F_1 and F_1 as measured from the F_2 positions would be the same if these special infinitesimal light-clocks are used. If noninfinitesimal N-world light-clocks are used, then the values will be approximately the same and equal in the limit.

Expression 6.4 relates the light-clock counts relative to the measure of the to-and-fro paths of light transmission. By not substituting for $x_{Ea}^{(2)}$, it is easily seen that $x_{Ea}^{(2)} \approx LG$, where G is an expression written entirely in terms of various light-clock count numbers. This implies that the so-called 4-tuples $(\mathtt{st}(x_{Ea}^{(1)}), \mathtt{st}(y_{Ea}^{(1)}), \mathtt{st}(z_{Ea}^{(1)}), \mathtt{st}(t_{Ea}^{(1}))$, $(\mathtt{st}(x_{Ea}^{(2)}), \mathtt{st}(y_{Ea}^{(2)}), \mathtt{st}(z_{Ea}^{(2)}), \mathtt{st}(t_{Ea}^{(2)}))$ are not the absolute Cartesian type coordinates determined by Euclidean geometry and used to model Galilean dynamics. These coordinates are dynamically determined by the behavior of electromagnetic radiation within the N-world. Indeed, in [7], the analysis within the (outside of the monadic clusters) that leads to Prokhovnik's conclusions is only relative to electromagnetic propagation and is done by pure number Galilean dynamics. Recall that the monadic cluster analysis is also done by Galilean dynamics.

In general, when it is claimed that "length contracts" with respect to relative velocity the "proof" is stated as follows: $x' = \mathtt{st}(\beta)(x + vt)$; $\overline{x}' = \mathtt{st}(\beta)(\overline{x} + \overline{v}\overline{t})$. Then these two expressions are subtracted. Supposedly, this yields $\overline{x}' - x' = \mathtt{st}(\beta)(\overline{x} - x)$ since its assumed that $\overline{v}\overline{t} = vt$. For defined coordinates $\overline{x}_E^{(i)},\ x_E^{(i)},\ i = 1, 2$, a more complete expression would be

$$\overline{x}_E^{(1)} - x_E^{(1)} = \mathtt{st}(\beta)((\overline{x}_E^{(2)} - x_E^{(2)}) + (\overline{v}_E^{(3)}\overline{t}_E^{(2)} - v_E^{(3)}t_E^{(2)})). \tag{6.7}$$

In this particular analysis, it has been assumed that all NSP-world relative velocities $\omega_i, \overline{\omega_i} \geq 0$. To obtain the classical length contraction expression, let $\omega_i = \overline{\omega_i},\ i = 1, 2, 3$. Now this implies that $\overline{\theta} = \theta,\ \overline{\phi} = \phi$ as they appear in the velocity figure on page 52 and that

$$\overline{x}_E^{(1)} - x_E^{(1)} = \mathtt{st}(\beta)(\overline{x}_E^{(2)} - x_E^{(2)}). \tag{6.8}$$

The difficulty with this expression has been its interpretation. Many modern treatments of Special Relativity [6] argue that (6.8) has no physical meaning. But in these arguments it is assumed that $\overline{x}_E^{(1)} - x_E^{(1)}$ means "length" in the Cartesian coordinate sense as related to Galilean dynamics. As pointed out, such a physical meaning is not the case. Expression (6.8) is a relationship between light-clock counts and, in general, displays properties of electromagnetic propagation within the N-world. Is there a difference between the right and left-hand sides of 6.8 when viewed entirely from the NSP-world. First, express 6.8 as $\overline{x}_E^{(1)} - x_E^{(1)} = \mathtt{st}(\beta)\overline{x}_E^{(2)} - \mathtt{st}(\beta)x_E^{(2)}$. In terms of operational light-clock counts, this expression becomes

$$L(\overline{\lambda}^{(1)}\, {}^*\!\cos\theta - \overline{\eta}^{(1)}\, {}^*\!\cos\theta) - L(\lambda^{(1)}\, {}^*\!\cos\theta - \eta^{(1)}\, {}^*\!\cos\theta) \approx \tag{6.9}$$

$$L(\overline{\lambda}^{(2)}\beta|\, {}^*\!\cos\phi| - \overline{\eta}^{(2)}\beta|\, {}^*\!\cos\phi|) - L(\lambda^{(2)}\beta|\, {}^*\!\cos\phi| - \eta^{(2)}\beta|\, {}^*\!\cos\phi|),$$

where finite $\beta = (1 - (K^{(3)})^2)^{-1/2}$ and $|\cdot|$ is used so that the Einstein velocities are not directed numbers and the Einstein distances are comparable. Also as long as $\theta,\ \phi$ satisfy the velocity figure on page 45, then (6.9) is independent of the specific angles chosen in the N-world since in the N-world expression (6.8) no angles appear relating the relative velocities. That is, the velocities are not vector quantities in the N-world, but scalars.

Assuming the nontrivial case that $\theta \not\approx \pi/2,\ \phi \not\approx \pi$, we have from Theorem 11.1.1 [9] that there exist $\overline{\Lambda}^{(i)},\ \overline{N}^{(i)},\ \Lambda^{(i)},\ N^{(i)} \in \mathbb{N}_\infty,\ i = 1, 2$ such that ${}^*\!\cos\theta \approx \overline{\Lambda}^{(1)}/\overline{\lambda}^{(1)} \approx \overline{N}^{(1)}/\overline{\eta}^{(1)} \approx \Lambda^{(1)}/\lambda^{(1)} \approx N^{(1)}/\eta^{(1)},\ \beta|{}^*\!\cos\phi| \approx \overline{\Lambda}^{(2)}/\overline{\lambda}^{(2)} \approx \overline{N}^{(2)}/\overline{\eta}^{(2)} \approx \Lambda^{(2)}/\lambda^{(2)} \approx N^{(1)}/\eta^{(2)}$. Consequently, using the finite character of these quotients and the finite character of $L(\overline{\lambda}^{(i)}),\ L(\overline{\eta}^{(i)}),\ L(\lambda^{(i)}),\ L(\eta^{(i)}),\ i = 1, 2$, the general three body NSP-world view 6.9 is

$$L(\overline{\Lambda}^{(1)} - \overline{N}^{(1)}) - L(\Lambda^{(1)} - N^{(1)}) = L\Gamma^{(1)} \approx$$
$$L\Gamma_1^{(2)} = L(\overline{\Lambda}^{(2)} - \overline{N}^{(2)}) - L(\Lambda^{(2)} - N^{(2)}). \tag{6.10}$$

The obvious interpretation of 6.10 from the simple NSP-world light propagation viewpoint is displayed by taking the standard part of expression 6.10.

$$\mathtt{st}(L(\overline{\Lambda}^{(1)} - \overline{N}^{(1)})) - \mathtt{st}(L(\Lambda^{(1)} - N^{(1)})) = \mathtt{st}(L\Gamma^{(1)}) =$$
$$\mathtt{st}(L\Gamma_1^{(2)}) = \mathtt{st}(L(\overline{\Lambda}^{(1)} - \overline{N}^{(1)})) - \mathtt{st}(L(\Lambda^{(1)} - N^{(1)})). \tag{6.11}$$

This is the general view as to the equality of the standard NSP-world distance traveled by a light pulse moving to-and-fro within a light-clock as used to measure at F_1 and F_2, as viewed from the NSPPM only, the occurrence of the light pulse event from P. In order to interpret 6.9 for the N-world and a single NSP-world relative velocity, you consider additionally that $\omega_1 = \omega_2 = \omega_3$. Hence, $\theta = \pi/3$ and correspondingly $\phi = 2\pi/3$. In this case, β is unaltered and since $\cos\pi/3,\ \cos 2\pi/3$ are nonzero and finite, 6.9 now yields

$$\mathtt{st}(L(\overline{\lambda}^{(1)} - \overline{\eta}^{(1)})) - \mathtt{st}(L(\lambda^{(1)} - \eta^{(1)})) =$$
$$\mathtt{st}(\beta)(\mathtt{st}(L(\overline{\lambda}_1^{(2)} - \overline{\eta}_1^{(2)})) - \mathtt{st}(L(\lambda_1^{(2)} - \eta_1^{(2)}))) \Rightarrow$$
$$(\mathtt{st}(L\overline{\lambda}^{(1)}) - \mathtt{st}(L\overline{\eta}^{(1)})) - (\mathtt{st}(L\lambda^{(1)}) - \mathtt{st}(L\eta^{(1)})) =$$
$$\mathtt{st}(\beta)((\mathtt{st}(L\overline{\lambda}_1^{(2)}) - \mathtt{st}(L\overline{\eta}_1^{(2)})) - (\mathtt{st}(L\lambda_1^{(2)}) - \mathtt{st}(L\eta_1^{(2)}))). \tag{6.12}$$

Or

$$\mathtt{st}(L(\overline{\lambda}^{(1)} - \overline{\eta}^{(1)}) - L(\lambda^{(1)} - \eta^{(1)})) =$$
$$\mathtt{st}(L[(\overline{\lambda}^{(1)} - \overline{\eta}^{(1)}) - (\lambda^{(1)} - \eta^{(1)})]) =$$
$$\mathtt{st}(L\Pi^{(1)}) = \mathtt{st}(\beta)\mathtt{st}(L\Pi_1^{(2)}) = \mathtt{st}(\beta L\Pi_1^{(2)}) = \tag{6.13}$$
$$\mathtt{st}(L[(\overline{\lambda}^{(1)} - \overline{\eta}^{(1)}) - (\lambda^{(1)} - \eta^{(1)})]) =$$
$$\mathtt{st}(\beta L[(\overline{\lambda}_1^{(2)} - \overline{\eta}_1^{(2)}) - (\lambda_1^{(2)} - \eta_1^{(2)})]).$$

In order to obtain the so-called "time dilation" expressions, follow the same procedure as above. Notice, however, that (6.3) leads to a contradiction unless

$$u((\overline{\lambda}^{(1)} + \overline{\eta}^{(1)}) - (\lambda^{(1)} + \eta^{(1)})) \approx \beta u((\overline{\lambda}^{(2)} + \overline{\eta}^{(2)}) - (\lambda^{(2)} + \eta^{(2)})). \tag{6.14}$$

It is interesting, but not surprising, that this procedure yields (6.14) without hypothesizing a relation between the $\omega_i,\ i = 1, 2, 3$ and implies that the timing infinitesimal light-clocks are the fundamental constitutes for the analysis. In the NSP-world, 6.14 can be re-expressed as

$$u((\overline{\lambda}^{(1)} + \overline{\eta}^{(1)}) - (\lambda^{(1)} + \eta^{(1)})) \approx u(\overline{\lambda}_2^{(2)} - \lambda_2^{(2)}). \tag{6.15}$$

Or

$$\mathtt{st}(u((\overline{\lambda}^{(1)} + \overline{\eta}^{(1)})) = \mathtt{st}(u\Pi_2^{(1)}) =$$
$$\mathtt{st}(u\Pi_3^{(2)}) = \mathtt{st}(u(\overline{\lambda}_2^{(2)} - \lambda_2^{(2)})). \tag{6.16}$$

[See note 4.] From the N-world, the expression becomes, taking the standard part operator,

$$\mathtt{st}(u(\overline{\lambda}^{(1)} + \overline{\eta}^{(1)})) - \mathtt{st}(u(\lambda^{(1)} + \eta^{(1)})) =$$
$$\mathtt{st}(\beta)(\mathtt{st}(u(\overline{\lambda}^{(2)} + \overline{\eta}^{(2)})) - \mathtt{st}(u(\lambda^{(2)} + \eta^{(2)}))). \tag{6.17}$$

Or

$$\mathtt{st}(u\Pi_2^{(1)}) = \mathtt{st}(\beta)\mathtt{st}(u\Pi_4^{(2)}) = \mathtt{st}(\beta u\Pi_4^{(2)}) =$$
$$\mathtt{st}(u((\overline{\lambda}^{(1)} + \overline{\eta}^{(1)}) - (\lambda^{(1)} + \eta^{(1)}))) = \mathtt{st}(\beta u[(\overline{\lambda}^{(2)} + \overline{\eta}^{(2)}) - (\lambda^{(2)} + \eta^{(2)})]). \tag{6.18}$$

Note that using the standard part operator in the above expressions, yields continuum time and space coordinates to which the calculus can now be applied. However, the time and space measurements are not to be made with respect to an universal (absolute) clock or ruler. The measurements are relative to electromagnetic propagation. The Einstein time and length are not the NSPPM time and length, but rather they are concepts that incorporate a mode of measurement into electromagnetic field theory. This mode of measurement follows from the one wave property used for Special Theory scenarios, the property that, in the N-world, the propagation of a photon do not take on the velocity of its source. It is this that helps clarify properties of the NSPPM. Expressions such as (6.13), (6.18) will be interpreted in the next sections of this paper.

7. An Interpretation.

In each of the expressions $(6.i)$, $i = 10, \ldots, 18$ the infinitesimal numbers L, u are unaltered. If this is the case, then the light-clock counts would appear to be altered. As shown in Note [2], alteration of c can be represented as alterations that yield infinite counts. Thus, in one case, you have a specific infinitesimal L and for the other infinitesimal light-clocks a different light-clock c is used. But, $L/u = c$. Consequently the only alteration that takes place in N-world expressions $(6.i)$, $i = 12, 13, 17, 18$ is the infiniteimal light-clocks that need to be employed. This is exactly what (6.13) and (6.18) state if you consider it written as say, $(\beta L)\cdot$ rather than $L(\beta\ \cdot)$. Although these are external expressions and cannot be "formally" transferred back to the N-world, the methods of infinitesimal modeling require the concepts of "constant" and "not constant" to be preserved.

These N-world expressions can be re-described in terms of N-world approximations. Simply substitute $\doteq$ for $=$, a nonzero real d [resp. μ] for L [resp. u] and real natural numbers for each light-clock count in equations $(6.i)$, $i = 12, 17$. Then for a particular d [resp. μ] any change in the light-clock measured relative velocity v_E would dictate a change in the the light-clocks used. Hence, the N-world need not be concerned with the idea that "length" contracts but rather it is the required light-clocks change. It is the required change in infiniteimal light-clocks that lead to real physical changes in behavior as such behavior is compared to a standard behavior. *But, in many cases, the use of light-clocks is not intended to be a literal use of such instruments.* For certain scenarios, light-clocks are to be considered as *analog models* that incorporate electromagnetic energy properties. [See note 18, first paragraph.]

The analysis given in the section 3 is done to discover a general property for the transmission of electromagnetic radiation. It is clear that property (*) does not require that the measured velocity of light be a universal constant. All that is needed is that for the two NSP-world times $t_a,\ t_b$ that $\mathtt{st}(\ell(t_a)) = \mathtt{st}(\ell_1(t_b))$. This means that all that is required for the most basic aspects of the Special Theory to hold is that at two NSP-world times in the $F_1 \to F_2,\ F_2 \to F_1$ reflection process $\mathtt{st}(\ell(t_a)) = \mathtt{st}(\ell_1(t_b))$, t_a a time during the transmission prior to reflection and t_b after reflection. If $\ell,\ \ell_1$ are nonstandard extensions of standard functions $v,\ v_1$ continuous on $[a,b]$, then given any $\epsilon \in \mathbb{R}^+$ there is a δ such that for each $t,\ t' \in [a,b]$ such that $|t-t'| < \delta$ it follows that $|v(t)-v(t')| < \epsilon/3$ and $|v_1(t)-v_1(t')| < \epsilon/3$. Letting $t_3 - t_1 < \delta$, then $|t_a - t_b| < \delta$. Since ${}^*v(t_a) = \ell(t_a) \approx {}^*v_1(t_b) = \ell_1(t_b)$, *-transfer implies $|{}^*v(t_2) - {}^*v_1(t_2)| < \epsilon$. [See note 5.] Since t_2 is a standard number, $|v(t_2) - v_1(t_2)| < \epsilon$ implies that $v(t_2) = v_1(t_2)$. Hence, in this case, the two functions $\ell,\ \ell_1$ do not differentiate between the velocity c at t_2. But t_2 can be considered an arbitrary (i.e. NSPPM) time such that $t_1 < t_2 < t_3$. **This does not require c to be the same for all cosmic times** only that $v(t) = v_1(t),\ t_1 < t < t_3$.

The restriction that $\ell,\ \ell_1$ are extended standard functions appears necessary for our derivation. Also, this analysis is not related to what ℓ may be for a stationary laboratory. In the case of stationary $F_1,\ F_2$, then the integrals are zero in equation (19) of section 3. The easiest thing to do is to simply postulate that $\mathtt{st}({}^*v(t_a))$ is a universal constant. This does not make such an assumption correct.

One of the properties that will allow the Einstein velocity transformation expression to be derived is the *equilinear* property. **This property is weaker than the $c =$ constant property for light propagation.** Suppose that you have within the NSP-world three observers $F_1,\ F_2,\ F_3$ that are linearly related. Further, suppose that w_1 is the NSP-world velocity of F_2 relative to F_1 and w_2 is the NSP-world velocity of F_3 relative to F_2. It is assumed that for this nonmonadic cluster situation, that Galilean dynamics also apply and that $\mathtt{st}(w_1) + \mathtt{st}(w_2) = \mathtt{st}(w_3)$. Using the description for light propagation as given in section 3, let t_1 be the cosmic time when a light pulse leaves F_1, t_2 when it "passes" F_2, and t_3 the cosmic time when it arrives at F_3.

From equation (3.15), it follows that

$$\mathtt{st}(w_1) = \mathtt{st}({}^*v_1(t_{1a}))\mathtt{st}\left({}^*\!\!\int_{t_1}^{t_2} \frac{1}{x}dx\right) +$$

$$[\mathtt{st}(w_2) =]\mathtt{st}({}^*v_2(t_{2a}))\mathtt{st}\left({}^*\!\!\int_{t_2}^{t_3} \frac{1}{x}dx\right) =$$

$$\mathtt{st}(w_3) = \mathtt{st}({}^*v_3(t_{3a}))\mathtt{st}\left({}^*\!\!\int_{t_1}^{t_3} \frac{1}{x}dx\right). \tag{7.1}$$

If $\mathtt{st}({}^*v_1(t_{1a})) = \mathtt{st}({}^*v_2(t_{2a})) = \mathtt{st}({}^*v_3(t_{3a}))$, then we say that the velocity functions ${}^*v_1,\ {}^*v_2,\ {}^*v_3$ are *equilinear.* The constancy of c implies equilinear, but not conversely. In either case, functions such as ${}^*v_1$ and ${}^*v_2$ need not be the same within a stationary laboratory after interaction.

Experimentation indicates that electromagnetic propagation does "appear" to behave in the N-world in such a way that it does not accquire the velocity of the source. The light-clock analysis is consistent with the following speculation. **Depending upon the scenario, the uniform velocity yields an effect via interactions with the subparticle field (the NSPPM) that uses a photon particle behavioral model. This is termed the (emis) effect.** Recall that a "light-clock" can be considered as an analog model for the most basic of the electromagnetic properties. On the other hand, only those experimental methods that replicate or are equivalent to the methods of Einstein measure would be relative to the Special Theory. This is one of the basic logical errors in theory application. The experimental language must be related to the language of the derivation. The concept of the light-clock, linear paths and the like are all intended to imply NSPPM interactions. Any explanation for experimentally verified Special Theory effects should be stated in such a language and none other. I also point out that there are no paradoxes in this derivation for you cannot simply "change your mind" with respect to the NSPPM. For example, an observer is either in motion or not in motion, and not both **with respect to the NSPPM.**

8. A Speculation and Ambiguous Interpretations

Suppose that the correct principles of infinitesimal modeling were known prior to the M-M (i.e. Michelson-Morley) experiment. Scientists would know that the (mathematical) NSPPM is not an N-world entity. They would know that they could have very little knowledge as to the refined workings of this NSP-world NSPPM since $\approx$ is not an $=$. They would have been forced to accept the statement of Max Planck that "Nature does not allow herself to be exhaustively expressed in human thought."[*The Mechanics of Deformable Bodies*, *Vol. II*, *Introduction to Theoretical Physics*, Macmillian, N.Y. (1932),p. 2.]

Further suppose, that human comprehension was advanced enough so that all scientific experimentation always included a theory of measurement. The M-M experiment would then have been performed to learn, if possible, more about this NSP-world NSPPM. When a null finding was obtained then a derivation such as that in section 3 might have been forthcoming. Then the following two expressions would have emerged from the derivation.

The Einstein method for measurement - the "radar" method - is used (see A3, p. 52) to determining the relative velocity of the moving light-clock. Using Appendix-A equations (A14), let P correspond to F_2. Then $\theta = 0,\ \phi = \pi/2$. Since, $x^{(2)} = 0$ from page 54, then F_2 is the s-point Hence, $t_E^2 = t^{(2)}$. The superscript and subscript s represents local measurements about the s-point, using various devices, for laboratory standards (i.e. standard behavior) and using infinitesimal light-clocks or approximating devices such as atomic-clocks. [Due to their construction atomic clocks are effected by relativistic motion and gravitational fields approximately as the infinitesimal light-clock's counts are effected.] Superscript or subscript m indicates local measurements, using the same devices, for an entity considered at the m-point in motion relative to the s-point, where Einstein time and distance via the radar method as registered at s are used to investigate m-point behavior. For example, m-point time is measured at the s-point via infinitesimal light clock and the radar method and this represents time at the m-point. To determine how physical behavior is being altered, the m and s-measurements are compared. Many claim that you can replace each s with m, and m with s in what follows. This may lead to various controversies which are elimianted in part 3. A specific interpretation of

$$\mathtt{st}(\beta)^{-1}(\overline{t}^{(s)} - t^{(s)}) = \overline{t}_E^{(m)} - t_E^{(m)} \tag{8.1}$$

or the corresponding

$$\mathtt{st}(\beta)^{-1}(\overline{x}^{(s)} - x^{(s)}) = \overline{x}_E^{(m)} - x_E^{(m)} \tag{8.2}$$

seems necessary. However, (8.2) is unnecessary since $v_E(\mathtt{st}(\beta)^{-1}(\overline{t}^{(s)} - t^{(s)})) = v_E(\overline{t}_E^{(m)} - t_E^{(m)})$ yields (8.2), which can be used when convienient. Thus, only the infinitesimal light-clock "time" alterations are significant. Actual length as measured via the radar method is not altered. It is the clock counts that are altered.

<u>If</u>, in (8.2), which is employed for convenience, $\overline{x}^{(s)} - x^{(s)} = U^s$ (note that $x^{(s)} = v_E t^{(s)}$ etc.) is interpreted as "any" standard unit for length measurement at the s-point and $\overline{x}_E^{(m)} - x_E^{(m)}) = U^m$ the same "standard" unit for length measurement in a system moving with respect to the NSPPM

(without regard to direction), then for equality to take place the unit of measure U^m may seem to be altered in the moving system. Of course, it would have been immediately realized that the error in this last statement is that U^s is "any" unit of measure. Once again, the error in these two statements is the term "any." (This problem is removed by application of $(14)_a$ or $(14)_b$ p. 60.)

If, in (8.2), which is employed for convenience, $\overline{x}^{(s)} - x^{(s)} = U^s$ (Note that $x^{(s)} = v_E t^{(s)}$.) is interpreted as "any" standard unit for length measurement at the s-point and $\overline{x}_E^{(m)} - x_E^{(m)}) = U^m$ the same "standard" unit for length measurement in a system moving with respect to the NSPPM (without regard to direction), then for equality to take place the unit of measure U^m may seem to be altered in the moving system. Of course, it would have been immediately realized that the error in this last statement is that U^s is "any" unit of measure. Once again, the error in these two statements is the term "any." (This problem is removed by application of $(14)_a$ or $(14)_b$ p. 60.)

Consider experiements such as the M-M, Kennedy-Thorndike and many others. When viewed from the wave state, the interferometer measurement technique is determined completely by a light-clock type process – the number of light waves in the linear path. We need to use L^m_{sc}, a scenario associated light unit, for U^m and use a L^s_{sc} for U^s. It appears for this particular scenario, that L^s_{sc} may be considered the private unit of length in the NSP-world, such as L, used to measure NSP-world light-path length. The "wavelength" λ of any light source must also be measured in the same light units. Let $\lambda = N^s L^s_{sc}$. Taking into consideration a unit conversion factor k between the unknown NSP-world private units, such that $\mathtt{st}(kL^s_{sc}) = U^s$, the number of light waves in s-laboratory would be $A^s\mathtt{st}(kL^s_{sc})/N^s\mathtt{st}(kL^s_{sc}) = A^s/N^s$, where A^s is a pure number such that $A^s\mathtt{st}(kL^s_{sc})$ is the "path-length" using the units in the s-system. In the moving system, assuming that this simple aspect of light propagation holds in the NSP-world and the N-world which we did to obtain the derivation in section 3, it is claimed that substitution yields $\mathtt{st}(\beta^{-1}A^s kL^s_{sc})/\mathtt{st}(\beta^{-1}N^s kL^s_{sc}) = A^s\mathtt{st}(\beta^{-1}kL^s_{sc})/N^s\mathtt{st}(\beta^{-1}kL^s_{sc}) = A^s/N^s$. Thus there would be no difference in the number of light waves in any case where the experimental set up involved the sum of light paths each of which corresponds to the to-and-fro process [1: 24]. Further, the same conclusions would be reached using (8.2). not relevant to a Sagnac type of experiment. However, this does not mean that a similar derivation involving a polygonal propagation path cannot be obtained. [Indeed, this may be a consequence of a result to be derived in article 3. However, see note 8 part 4, p. 80.]

Where is the logical error in the above argument? The error is the object upon which the $\mathtt{st}(\beta)^{-1}$ operates. Specifically (6.13) states that

$$\mathtt{st}(\beta)^{-1}(A^s kL^s_{sc}) \overset{\text{(emis)}}{\longleftrightarrow} \beta^{-1}(L\Pi^{(s)}) = (\beta^{-1}L)\Pi^{(s)} \text{ and} \tag{8.3}$$

$$\mathtt{st}(\beta)^{-1}(N^s kL^s_{sc}) \overset{\text{(emis)}}{\longleftrightarrow} \beta^{-1}(L\Pi^{(s)}_1) = (\beta^{-1}L)\Pi^{(s)}_1. \tag{8.4}$$

It is now rather obvious that the two (emis) aspects of the M-M experiment nullify each other. Also for no finite w can $\beta \approx 0$. There is a great difference between the propagation properties in the NSP-world and the N-world. For example, the classical Doppler effect is an N-world effect relative to linear propagation. **Rather than indicating that the NSPPM is not present**, **the M-M results indicate indirectly that the NSP-world NSPPM exists.**

Apparently, the well-known Ives-Stillwell, and all similar, experiments used in an attempt to verify such things as the relativistic redshift are of such a nature that they eliminate other effects that motion is assumed to have upon the scenario associated electromagnetic *propagation.* What was shown is that the frequency ν of the canal rays vary with respect to a representation for v_E measured from electromagnetic theory in the form $\nu_m = \mathtt{st}(\beta)^{-1}\nu_s$. First, we must investigate what the so-called time dilation statement (8.2) means. What it means is exemplified by (6.14) and how the human mind comprehends the measure of "time." In the scenario associated (8.2) expression, for the right and left-sides to be comprehensible, the expression should be conceived of as a measure that originates with infinitesimal light-clock behavior. It is the experience with a specific unit and the number of them that "passes" that yields the intuitive concept of "observer time." On the other hand, for some purposes or as some authors assume, (8.2) might be viewed as a change in a time unit T^s rather than in an infinitesimal light-clock. Both of these interpretations can be incorporated into a frequency statement. First, relative to the frequency of light-clock counts, for a fixed stationary unit of time T^s, (8.2) reads

$$\mathtt{st}(\beta)^{-1}C^s_{sc}/T^s \doteq C^m_{sc}/T^s \Rightarrow \mathtt{st}(\beta)^{-1}C^s_{sc} \doteq C^m_{sc}. \tag{8.5}$$

But according to (6.18), the C^s_{sc} and C^m_{sc} correspond to infinitesimal light-clocks measures and nothing more than that. Indeed, (8.5) has nothing to do with the concept of absolute "time" only with the different infinitesimal light-clocks that need to be used due to relative motion. This requirement may be due to (emis). Indeed, the "length contraction" expression (8.1) and the "time dilation" expression (8.2) have nothing to do with either absolute length or absolute time. These two expressions are both saying the same thing from two different viewpoints. There is an alteration due to the (emis). [Note that the second $\doteq$ in (8.5) depends upon the T^s chosen.]

On the other hand, for a relativistic redshift type experiment, the usual interpretation is that $\nu_s \doteq p/T^s$ and $\nu_m \doteq p/T^m$. This leads to $p/T^m \doteq \mathtt{st}(\beta)^{-1}p/T^s \Rightarrow T^m \doteq \mathtt{st}(\beta)T^s$. Assuming that all frequency alterations due to (emis) have been eliminated then this is interpreted to mean that "time" is slower in the moving excited hydrogen atom than in the "stationary" laboratory. When compared to (8.5), there is the ambiguous interpretation in that the p is considered the same for both sides (i.e. the concept of the frequency is not altered by NSPPM motion). It is consistent with all that has come before that the Ives-Stillwell result be written as $\nu_s \doteq p/T^s$ and that $\nu_m \doteq q/T^s$, where "time" as a general notion is not altered. This leads to the expression

$$\mathtt{st}(\beta)^{-1}p \doteq q \ [= \text{ in the limit}]. \tag{8.6}$$

Expression (8.6) does not correspond to a concept of "time" but rather to the concept of alterations in emitted frequency due to (emis). One, therefore, has an ambiguous interpretation that in an Ives-Stillwell scenario the number that represents the frequency of light emitted from an atomic unit moving with velocity ω with respect to the NSPPM is altered due to (emis). This (emis) alteration depends upon $K^{(3)}$. It is critical that the two different infinitesimal light-clock interpretations be understood. One interpretation is relative to electromagnetic *propagation* theory. In this case, the light-clock concept is taken in its most literal form. The second interpretation is relative to an infinitesimal light-clock as an *analogue* model. This means that the cause need not be related to propagation but is more probably due to how individual constituents interact with the NSPPM. The exact nature of this interaction and a non-ambiguous approach needs further investigation based upon constituent models since the analogue model specifically denies that there is some type of *absolute time* dilation but, rather, signifies the existences of other possible causes. [In article 3, the $\nu_m = \mathtt{st}(\beta)^{-1}\nu_s$ is formally and non-ambiguously derived from a special line-element, a universal functional requirement and Schrödinger's equation.]

It is clear, however, that under our assumption that the scalar velocities in the NSP-world are additive with respect to linear motion, then if F_1 has a velocity ω with respect to the NSPPM and

F_2 has the velocity ω', then it follows that the light-clock counts for F_1 require the use of a different light-clock with respect to a stationary F_0 due to the (emis) and the light-clocks for F_2 have been similarly changed with respect to a stationary F_0 due to (emis). Consequently, a light-clock related expressed by $K^{(3)}$ is the result of the combination, so to speak, of these two (emis) influences. The relative NSPPM velocity ω_2 of F_1 with respect to F_2 which yields the difference between these influences is that which would satisfies the additive rule for three linear positions.

As previously stated, within the NSP-world relative to electromagnetic propagation, observer scalar velocities are either additive or related as discussed above. Within the N-world, this last statement need not be so. Velocities of individual entities are modeled by either vectors or, at the least, by signed numbers. Once the N-world expression is developed, then it can be modified in accordance with the usual (emis) alterations, in which case the velocity statements are N-world Einstein measures. For example, deriving the so-called relativistic Dopplertarian effect, the combination of the classical and the relativistic redshift, by means of a NSPPM argument such as appears in [7] where it is assumed that the light propagation laws with respect to the photon concept in the NSP-world are the same as those in the N-world, is in logical error. Deriving the classical Doppler effect expression then, when physically justified, making the wave number alteration in accordance with the (emis) would be the correct logic needed to obtain the relativistic Dopplertarian effect. [See note 6.]

Although I will not, as yet, re-interpreted Special Relativity results with respect to this purely electromagnetic interpretation, it is interesting to note the following two re-interpretations. The so-called variation of "mass" was, in truth, originally derived for imponderable matter (i.e. elementary matter.) This would lead one to believe that the so-called rest mass and its alteration, if experimentally verified, is really a manifestation of the electromagnetic nature of such elementary matter. Once again the so-called mass alteration can be associated with an (emis) concept. The μ-meson decay rate may also show the same type of alteration as appears to be the case in an Ives-Stillwell experiment. It does not take a great stretch of the imagination to again attribute the apparent alteration in this rate to an (emis) process. This would lead to the possibility that such decay is controlled by electromagnetic properties. Indeed, in order to conserve various things, μ-meson decay is said to lead to the generation of the neutrino and antineutrino. [After this paper was completed, a method was discovered that establishes that predicted mass and decay time alterations are (emis) effects. The derivations are found in article 3.]

I note that such things as neutrinos and antineutrinos need not exist. Indeed, the nonconservation of certain quantities for such a scenario leads to the conclusion that subparticles exist within the NSP-world and carry off the "missing" quantities. Thus the invention of such objects may definitely be considered as only a bookkeeping technique.

As pointed out, all such experimental verification of the properly interpreted transformation equations can be considered as indirect evidence that the NSP-world NSPPM exists. But none of these results should be extended beyond the experimental scenarios concerned. Furthermore, I conjecture that no matter how the human mind attempts to explain the (emis) in terms of a human language, it will always be necessary to postulate some interaction process with the NSPPM without being able to specifically describe this interaction in terms of more fundamental concepts. Finally, the MA-model specifically states that the Special Theory is a local theory and should not be extended, without careful consideration, beyond a local time interval $[a, b]$.

9. Reciprocal Relations

As is common to many mathematical models, not all relations generated by the mathematics need to correspond to physical reality. This is the modern approach to the length contradiction controversy [6]. Since this is a mathematical model, there is a theory of correspondence between the physical language and the mathematical structure. This correspondence should be retained throughout any derivation. This is a NSPPM theory and what is stationary or what is not stationary with respect to the NSPPM must be maintained throughout any correspondence. This applies to such reciprocal relations as

$$\mathtt{st}(\beta)^{-1}(\overline{t}_E^{(m)} - t_E^{(m)}) = \overline{t}^{(s)} - t^{(s)} \tag{9.1}$$

and

$$\mathtt{st}(\beta)^{-1}(\overline{x}_E^{(m)} - x_E^{(m)}) = \overline{x}^{(s)} - x^{(s)} \tag{9.2}$$

Statement (8.1) and (9.1) [resp. (8.2) and (9.2)] both hold from the NSPPM viewpoint only when $v_E = 0$ since it is not the question of the N-world viewpoint of relative velocity but rather the viewpoint that F_1 is fixed and F_2 is not fixed in the NSPPM or $\omega \leq \omega'$. The physical concept of the (s) and (m) must be maintained throughout the physical correspondence. Which expression would hold for a particular scenario depends upon laboratory confirmation. This is a scenario associated theory. All of the laboratory scenarios discussed in this paper use infinitesimalized (9.1) and (9.2) as derived from line-elements and the "view" or comparison is always made relative to the (s). Other authors, such as Dingle [1] and Builder [7], have, in a absolute sense, excepted one of these sets of equations, without derivation, rather the other set. I have not taken this stance in this paper.

One of the basic controversies associated with the Special Theory is whether (8.2) or (8.1) [resp. (9.1) or (9.2)] actually have physical meaning. The notion is that either "length" is a fundamental concept and "time" is defined in terms of it, or "time" is a fundamental concept and length is defined in terms of it. Ives, and many others assumed that the fundamental notion is length contraction and not time dilation. The modern approach is the opposite of this. Length contraction in the N-world has no physical meaning, but time dilation does [6]. We know that time is often defined in terms of length and velocities. But, the length or time being considered here is Einstein length or Einstein time. This is never mentioned when this problem is being considered. As discussed at the end of section 3, Einstein length is actually defined in terms of infinitesimal light-clocks or in terms of the Einstein velocity and Einstein time. As shown after equation (8.2) is considered, it is only infinitesimal light-clock "time" that is altered and length altertions is but a technical artefact. The changes in the infinitesimal light-clock counts yields an analogue model for physical changes that cause Special Theory effects. [See note 7.]

{Remark: Karl Popper notwithstanding, it is not the sole purpose of mathematical models to predict natural system behavior. The major purpose is to maintain logical rigor and, hopefully, <u>when applicable</u> to discover new properties for natural systems. I have used in this speculation a correspondence theory that takes the stance that any verifiable Special Theory effect is electromagnetic in character rather than a problem in measure. However, whether such effects are simply effects relative to the propagation of electromagnetic information or whether they are effects relative to the constituents involved cannot be directly obtain from the Special Theory. All mathematically stated effects involve the Einstein measure of relative velocity, v_E – a propagation related measure. The measure of an effect should also be done in accordance with electromagnetic theory. As demonstrated, the Special Theory should not be unnecessarily applied to the behavior of all nature systems since it is related to electromagnetic interaction; unless, of course, all natural systems are electromagnetic in character. Without strong justification, the assumption that one theory does apply to all scenarios is one of the greatest errors in mathematically modeling. But, if laboratory experiments verify that alterations are taking place in measured quantities and these variations are approximated in accordance with the Special Theory, then this would indicate that either the alterations are related to electromagnetic propagation properties or the constituents have an appropriate electromagnetic character.}

NOTES

[1] (a) Equation (3.9) is obtained as follows: since $t \in [a, b]$, t finite and not infinitesimal. Thus division by t preserves $\approx$. Hence,

$$\left[t\left(\frac{{}^*s(t+dt)-s(t)}{dt}\right)-s(t)\right]/t^2 \approx \frac{\ell(t)}{t}. \tag{1}$$

Since t is an arbitrary standard number and dt is assume to be an arbitrary and appropriate nonzero infinitesimal and the function $s(t)/t$ is differentiable, the standard part of the left-side equals the standard part of the right-side. Thus

$$\frac{d(s(t)/t)}{dt} = \frac{v(t)}{t}, \tag{2}$$

for each $t \in [a, b]$. By *-transfer, equation (3.9) holds for each $t \in {}^*[a, b]$.

(b) Equation (3.10) is then obtained by use of the *-integral and the fundamental theorem of integral calculus *-transferred to the NSP-world. It is useful to view the definite integral over a standard interval say $[t_1, t]$ as an operator, at least, defined on the set $C([t_1, t], \mathbb{R})$ of all continuous real valued functions defined on $[t_1, t]$. Thus, in general, the fundamental theorem of integral calculus can be viewed as the statement that $(f', f(t) - f(t_1)) \in \int_{t_1}^t$. Hence ${}^*(f', f(t) - f(t_1)) \in {}^*\int_{t_1}^t \Rightarrow ({}^*f', {}^*(f(t) - f(t_1))) \in {}^*\int_{t_1}^t \Rightarrow ({}^*f', f(t) - f(t_1)) \in {}^*\int_{t_1}^t$.

(c) To obtain the expressions in (3.19), consider $f(x) = 1/x$. Then ${}^*f$ is limited and S-continuous on ${}^*[a, b]$. Hence $({}^*f, \ln t_2 - \ln t_1) \in {}^*\int_{t_1}^{t_2}$. Hence $\mathtt{st}(({}^*f, \ln t_2 - \ln t_1)) = (f, \ln t_2 - \ln t_1) \in \int_{t_1}^{t_2}$. Further (3.19) can be interpreted as an interaction property.

[1.5] Infinitesimal light-clocks are based upon the QED model as to how electrons are kept in a range of distances in a hydrogen atom proton. The back-and-forth exchanges of photons between a proton and electron replaces "reflection" and the average distance between the proton and electron is infinitesimalized to the L. In this case, the proton and electron are also infinitesimalized. The large number of such interchanges over a second, in the model, is motivation for the use of the members of $\mathbb{N}_\infty^+$ as count numbers.

[2] The basic theorem that allows for the entire concept of infinitesimal light-clocks and the analysis that appears in this monograph has not been stated. As taken from "The Theory of Ultralogics,"the theorem, for this application, is:

Theorem 11.1.1 *Let* $10^\omega \in \mathbb{N}_\infty$. *Then for each* $r \in \mathbb{R}$ *there exists an* $x \in \{2m/10^\omega \mid (2m \in {}^*\mathbf{Z}) \wedge (|2m| < \lambda 10^\omega)\}$, *for any* $\lambda \in \mathbb{N}_\infty$, *such that* $x \approx r$ *(i.e.* $x \in \mu(r)$.*)*

Theorem 11.1.1 holds for other members of $\mathbb{N}_\infty$. Let $L = 1/10^\omega$ where ω is any hyperreal infinite natural number (i.e. $\omega \in \mathbb{N}_\infty$). Hence, by this theorem, for any positive real number r there exists some $m \in \mathbb{N}_\infty$ such that $2\mathtt{st}(m/10^\omega) = r$. I point out that for this nonzero case it is necessary that $m \in \mathbb{N}_\infty$ for if $m \in \mathbb{N}$, then $\mathtt{st}(m/10^\omega) = 0$. Since $c = \mathtt{st}(L/u)$, then $2\mathtt{st}(um) = 2\mathtt{st}((L/c)m) =$

$t = r/c$ as required. Thus, the infinitesimal light-clock determined length r and interval of time t are determined by the difference in infinitesimal light-clock counts $2m = (\lambda - \eta)$. Note that our approach allows the calculus to model this behavior by simply assuming that the standard functions are differentiable etc.

[2.5] (4 JUN 2000) Equating these counts here and elsewhere is done so that the "light pulse" is considered to have a "single instantaneous effect" from a global viewpoint and as such is not a signal in that globally it contains no information. Thus additional analysis is needed before one can state that the Special Theory applies to informational transmissions. It's obvious from section 7 that the actual value for c may depend upon the physical application of this theory.

[3] At this point and on, the subscripts on the τ have a different meaning than previously indicated. The subscripts denote process numbers while the superscript denotes the position numbers. For example, τ^2_{12} means the light-clock count number when the second light pulse leaves F_2 and τ^2_{31} would mean the light-clock count number when the first light pulse returns to position F_2.

The additional piece of each subscript denoted by the a on this and the following pages indicates, what I thought was obvious from the lines that follow their introduction, that these are approximating numbers that are infinitesimally near to standard NSP-world number obtained by taking the standard part.

[4] Note that such infinite hyperreal numbers as $\Pi^{(2)}_3$ (here and elsewhere) denote the difference between two infinitesimal light-clock counts and since we are excluding the finite number infinitesimally near to 0, these numbers must be infinite hyperreal. Infinitesimal light-clocks can be assumed to measure this number by use of a differential counter. BUT it is always to be conceived of as an infinitesimal light-clock "interval" (increment, difference, etc.) It is important to recall this when the various line-elements in the next article are considered.

[5] This result is obtained as follows: since $t_a \leq t_2 \leq t_b$, it follows that $|t_a - t_2| < \delta,\ |t_b - t_2| < \delta$. Hence by *-transfer, $|{}^*v(t_2) - {}^*v(t_a)| < \epsilon/3,\ |{}^*v_1(t_b) - {}^*v_1(t_2)| < \epsilon/3$. Since we assume arbitrary $\epsilon/3$ is a standard positive number, then ${}^*v(t_a) = \ell(t_a) \approx {}^*v_1(t_b) = \ell_1(t_b) \Rightarrow |{}^*v(t_a) - {}^*v_1(t_b)| < \epsilon/3$. Hence $|{}^*v(t_2) - {}^*v_1(t_2)| < \epsilon$.

[6] In this article, I mention that all previous derivations for the complete Dopplertarian effect (the N-world and the transverse) are in logical error. Although there are various reasons for a redshift not just the Dopplertarian, the electromagnetic redshift based solely upon properties of the NSPPM can be derived as follows:

(i) let ν^s denote the "standard" laboratory frequency for radiation emitted from an atomic system. This is usually determined by the observer. The NSP-world alteration in emitted frequency at an atomic structure due to (emis) is $\gamma\nu^s = \nu^{\text{radiation}}$, where $\gamma = \sqrt{1 - v_E^2/c^2}$ and v_E is the Einstein measure of the relative velocity using light-clocks only.

(ii) Assuming that an observer is observing this emitted radiation in a direct line with the propagation and the atomic structure is receding with velocity v from the observer, the frequency of the electromagnetic propagation, within the N-world, is altered compared to the observers standards. This alteration is $\nu^{\text{radiation}}(1/(1 + v/c)) = \nu^{\text{received}}$. Consequently, this yields the total alteration as $\gamma\nu^s(1/(1 + v/c)) = \nu^{\text{received}}$. Note that v is measured in the N-world and can be considered a directed velocity. Usually, if due to the fact that we are dealing with electromagnetic radiation, we consider v the Einstein measure of linear velocity (i.e. $v = v_E$), then the total Dopplertarian effect for $v \geq 0$ can be written as

$$\nu^s \left(\frac{1 - v_E/c}{1 + v_E/c}\right)^{1/2} = \nu^{\text{received}}. \tag{3}$$

It should always be remembered that there are other reasons, such as the gravitational redshift and others yet to be analyzed, that can mask this total Dopplertarian redshift.

[7] A question that has been asked relative to the new derivation that yields Special Theory resilts is why in the N-world do we have the apparent nonballistic effects associated with electromagnetic radiation? In the derivation, the opposite was assumed for the NSP-world monadic clusters. The constancy of the *measure*, by light-clocks and the like, of the $F_1 \to F_2,\ F_2 \to F_1$ velocity of electromagnetic radiation was modeled by letting $\mathtt{st}(t_a) = \mathtt{st}(t_b)$. As mentioned in the section on the Special Theory, the Einstein velocity measure transformation expression can be obtained prior to embedding the world into a hyperbolic velocity space. It is obtained by considering three in-line standard positions $F_1,\ F_2,\ F_3$ that have the NSP-world velocities w_1 for F_2 relative to F_1, w_2 for F_3 relative to F_2 and the simple composition $w_3 = w_1 + w_2$ for F_3 relative to F_1. Then simple substitution in this expression yields

$$v_E^{(3)} = (v_E^{(1)} + v_E^{(2)})/\left(1 + \frac{v_E^{(1)} v_E^{(2)}}{c^2}\right). \tag{4}$$

This relation is telling us something about the required behavior in the N-world of electromagnetic radiation. To see that within the N-world we need to assume for electromagnetic radiation effects the nonballistic property, simply let $v_E^{(2)} = c$ or $v_E^{(2)} \doteq c$. Then $v_E^{(3)} = c$, or $\doteq c$. Of course, the reason we do not have a contradiction is that we have two distinctly different views of the behavior of electromagnetic radiation, the NSP-world view and the N-world view. Further, note how, for consistency, the velocity of electromagnetic radiation is to be measured. It is measured by the Einstein method, or equivalent, relative to a to-and-fro path and measures of "time" and "distance" by means of a (infinitesimal) light-clock counts. Since one has the NSPPM, then letting F_1 be fixed in that medium, assuming that "absolute" physical standards are measured at F_1, equation (4) indicates why, in comparison, physical behavior varies at F_2 and F_3. The hyperbolic velocity space properties are the cause for such behavior differences.

I am convinced that the dual character of the Special theory derivation requires individual reflection in order to be understood fully. In the NSP-world, electromagnetic radiation behaves in one respect, at least, like a particle in that it satisfies the ballistic nature of particle motion. The reason that equation (3) is derivable is due to the definition of Einstein time. But *Einstein time, as measured by electromagnetic pulses, models the nonballistic or one and only one wave-like property in that a wave front does not partake of the velocity of the source.* This is the reason why I wrote that *a NSPPM disturbance would trace the same operational linear light-clock distance.* The measuring light-clocks are in the N-world in this case. F_1 is modeled as fixed in the NSPPM and F_2 has an NSP-world relative velocity. The instant the light pulse is reflected back to F_1 it does not, from the N-world viewpoint, partake of the N-world relative velocity and therefore traces out the exact same apparent N-world linear path. The position F_2 acts like a virtual position having no other N-world effect upon the light pulse except a reversal of direction.

[8] This expression implies that the "c" that appears here and elsewhere is to be measured by infinitesimal light-clocks. As noted $u \approx L/c$, but infinitesimal light-clock construction yields that $u = L/c$. For a fixed L, from the NSPPM viewpoint, u is fixed. Notice that $t^{(i)} \approx u(2\eta^{(i)}) = u(\gamma^{(i)}),\ \gamma^{(i)} \in {}^*\mathbb{N}_\infty^+$.

[9] In both parts of this monograph, conceptual time is used and NSPPM and gravitational field processes yield non-classical relations between these times. For example, $t_2 = \sqrt{t_1 t_3}$. For the Special Theory, there is only one aspect of physical-world behavior that corresponds to the infinitesimal-world behavior. This is the sudden photon interaction with other particles. Hence, such interactions are particle-like, which predicts the QED assumption. An NSPPM velocity for the source is always necessary for photon emission due to a photon's momentum. In the actual derivation, the wave-property, where classical wave-mechanics can be applied, is not a property within the infinitesimal-world. Classical wave-mechanics model photon paths of motion within our physical-world. Wave-behavior emerges after the "st" operator is applied. The particle behavior takes place only for the interactions. Hence, the probability interpretation that comes from a photon's wave-property can be used to predict the number of photon interactions. Consequently, there is neither a contradiction between these two interpretations nor the particle assumption.

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Overhead material relative to the paper "A corrected derivation for the Special Theory of relativity" as presented at the above mentioned MAA meeting of Nov. 14, 1992.

Relativity and Logical Error

In a 1922 lecture Einstein stated the bases of his Special Theory. "Time cannot be absolutely defined, and there is an inseparable relation between time and signal velocity." In the paper you're going to receive, the first phrase is shown to be **false**. Thus with respect to natural models, the stated hypotheses yield an inconsistent theory.

Originally, Einstein did NOT reject an ether or medium concept. In the same lecture, he said "Since then [1905] I have come to believe that the motion of the Earth [through the ether] cannot be detected by any optical experiment though the Earth is revolving about the Sun." Einstein also stated that he simply couldn't describe the properties of such an ether.

In his derivation, he first uses the term "clock" as meaning **any** measure of time within the natural world without further defining the apparatus. But then he restricts the characterization of such clocks by adding light propagation terminology relative to their synchronization and, hence, creates **a new predicate model**.

Einstein now uses these restricted clocks to measure a new time, **the proper time**, in terms of additional light propagation language. This is a third predicate model. After this, he <u>assumes</u> that the second predicate model is Newtonian infinitesimal time, another type of absolute time which is a different fourth predicate model. Thus substituting one predicate model for another, as if they are the same, he obtains the Lorentz transformation. Of course, this substitution is a logical error.

Now, Einstein's form of the Lorentz transformation has proper time on the left-side and Newtonian absolute time on the right. Then to apply these equations, the predicate model for proper time with its light propagation language is extended to include an absolute **any** time concept. The logical error of substituting one predicate model for another predicate is compounded by the error of **model theoretic generalization**. The statement that what holds for one domain (time restricted by the language of light propagation) cannot be extended ad hoc to a larger domain. . ..

Appendix-A

1. The Need for Hyperbolic Geometry

In this appendix, it is shown that from equations (3.21) and (3.22) the Lorentz transformation are derivable. All of the properties for the Special Theory are based upon "light" propagation. In Article 2, the concern is with two positions $F_1,\ F_2$ in the NSPPM within the NSP-world and how the proposed NSPPM influences such behavior. Prior to applications to the N-world, with the necessity for the N-world Einstein measures, the NSPPM exhibits infinitesimal behavior and special NSPPM non-classical global behavior. The behavior at specific moments of NSPPM time for global positions and classical uniform velocities are investigated.

The following is a classical description for photon behavior. Only NSPPM relative velocities (speeds) are being considered. Below is a global diagram for four points that began as the corners of a square, where u and ω denote uniform relative velocities between point locations and no other point velocities are considered. The meanings for the symbolized entities are discussed below.

$$\begin{array}{lll} & \bullet F_1\ t \leadsto\!\longrightarrow & \omega \longrightarrow \bullet F_2(t(p_1)) \leadsto\!\longrightarrow \\ & & \\ & \bullet F_1'\ t' \leadsto & \\ u & \downarrow & \\ & \bullet F_1'\ t(p_2) & \omega \longrightarrow \bullet F_2'(t(p_2)) \\ u & \downarrow & \downarrow\ u \end{array}$$

Consider the following sequence of (conceptual) NSPPM time-ordered events. First, the N-world position points $F_1,\ F_2,\ F_1',\ F_2'$ are stationary with respect to each other and form the corners of a very small rhombus, say the side-length is the average distance d between the electron and proton within an hydrogen atom. The sides are $\overline{F_1, F_2},\ \overline{F_2F_2'},\ \overline{F_2', F_1'},\ \overline{F_1'F_1}$. At the NSPPM time t_g, the almost coinciding F_2, F_2' uniformally recede from the almost coinciding F_1, F_1' with constant velocity ω. At a time $t > t_g$, where the distance between the two groups is significantly greater than d, one process occurs simultaneously. The point F_1' separates from F_1 with relative velocity u and F_2' separates from F_2 with a relative velocity u. [Using NSP-world processes, such simultaneity is possible relative to a non-photon transmission of information (Herrmann, 1999).] At any time $\geq t$, the elongating line segments $\overline{F_1F_1'}$ and $\overline{F_2F_2'}$ are parallel and they are not parallel to the parallel elongating line segments $\overline{F_1F_2}$ and $\overline{F_1'F_2'}$.

At NSPPM time t, a photon p_1 is emitted from F_1 towards F_2 and passes through F_2 and continues on. As F_1' recedes from F_1, at $t' > t$, a photon p_2 is emitted from F_1' towards F_2'. The original classical photon-particle property that within a monadic cluster photons prorogate with velocity $\omega + c$ is extended to this global environment. [Again there are NSP-world processes that can ensure that the emitted photons acquire this prorogation velocity (Herrmann, 1999).] Also, this classical photon-particle property is applied to u. Thus, photon p_2 is assumed to take on an additional velocity component u. Photon, p_1, passes through F_2 at the NSPPM time $t(p_1)$. Then p_2 is received at point F_2' at time $t(p_2)$.

Classically, $t(p_1') > t(p_1)$. From a viewpoint relative to elongating $\overline{F_1F_1'}$, the distance between the two photon-paths of motion measured parallel to elongating $\overline{F_1F_1'}$ is $u(t(p_2) - t)$. On the other hand, from the viewpoint of elongating $\overline{F_1F_1'}$, the distance between photon-paths, if they were parallel, is $u(t' - t)$. By the relativity principle, from the viewpoint of F_1', the first equation in (3.19) should apply. Integrating, where $\mathtt{st}({}^*v(t_a))) = c$, one obtains $u(t(p_2) - t)) = ue^{\omega/c}(t' - t)$. [Note: No reflection is required for this restricted application of (3.19).] This result is not the classical expression $u(t(p_2) - t))$. For better comprehension, use infinitesimal light-clocks to measure NSPPM time. Then using the same NSPPM process that yields information instantaneously throughout the standard portion of the NSPPM, all clocks used to determine these times can be set at zero when they indicate the time t. This yields that the two expressions for the distance are $ut(p_2)$ and $ue^{\omega/c}t'$. However, the classical expression $ut(p_2)$ has the time $t(p_2)$ dependent upon both ω and, after the t' moment, upon u. But, for the relativistic expression, the t' is neither dependent upon the u velocity after t' nor the ω and the factor $e^{\omega/c}$ has only one variable ω. What property does this NSPPM behavior have that differentiates it from the classical?

Consider the two velocities u and $ue^{\omega/c}$. These two velocities only correspond when $\omega = 0$. Hence, if we draw a velocity diagram, one would conclude that, in this case, the velocities are trivially "parallel." Using Lobatchewskian's horocycle construction, Kulczycki (1961) shows that for "parallel geometric" lines in hyperbolic space, the distance between each pair of such lines increases (or decreases) by a factor $e^{x/k}$, as one moves an ordinary distance x along the lines and k is some constant related to the x unit of measurement. Phrasing this in terms of velocities, where $x = \omega$ and $k = c$, then, for this case, the velocities, as represented in the NSPPM by standard real numbers, appear to satisfy the properties for an hyperbolic velocity-space. Such velocity behavior would lead to this non-classical NSPPM behavior.

When simple classical physics is applied to this simple Euclidian configuration within the NSPPM, then there is a transformation Φ: NSPPM $\to$ N-world, which is characterized by hyperbolic velocity-space properties. This is also the case for relative velocity and collinear points, which are exponentially related to the Einstein measure of relative velocity in the N-world. In what follows, this same example is used but generalized slightly by letting F_1 and F_2 coincide.

2. The Lorentz Transformations

Previously, we obtained the expression that $t_2 = \sqrt{t_1 t_3}$. The Einstein measures are defined formally as

$$\begin{cases} t_E = (1/2)(t_3 + t_1) \\ r_E = (1/2)c(t_3 - t_1) \\ v_E = r_E/t_E, \text{ where defined.} \end{cases} \quad (A1)$$

Notice that when $r_E = 0$, then $v_E = 0$ and $t_E = t_3 = t_1 = t_2$ is not Einstein measure.
The Einstein time t_E is obtained by considering the "flight-time" that would result from using one and only one wave-like property not part of the NSPPM but within the N-world. This property is that the c is not altered by the velocity of the source. This Einstein approach assumes that the light pulse path-length from F_1 to F_2 equals that from F_2 back to F_1. Thus, the Einstein flight-time used for the distance r_E is $(t_3 - t_1)/2$. The t_E, the Einstein time corresponding to an infinfinitesimal light-clock at F_2, satisfies $t_3 - t_E = t_E - t_1$. From $(A1)$, we have that

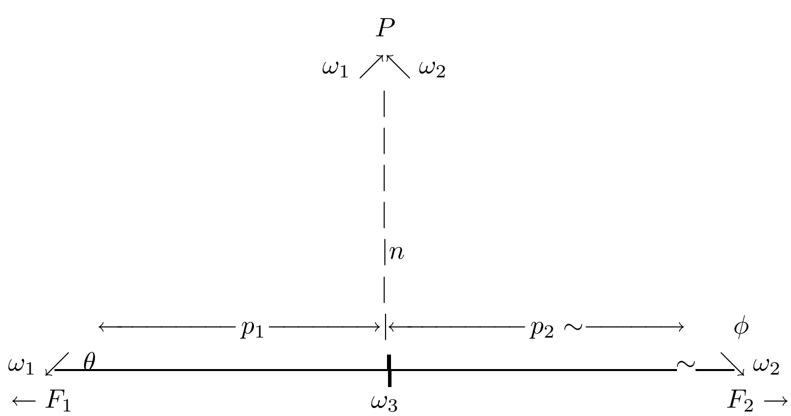
$$t_3 = (1 + v_E/c)t_E \text{ and } t_1 = (1 - v_E/c)t_E, \quad (A2)$$

and, hence, $t_2 = (\sqrt{1 - v_E^2/c^2})t_E$. Since $e^{\omega/c} = \sqrt{t_3/t_1}$, this yields

$$e^{\omega/c} = \left(\frac{1 + v_E/c}{1 - v_E/c}\right)^{(1/2)}. \quad (A3)$$

Although it would not be difficult to present all that comes next in terms of the nonstandard notions, it is not necessary since all of the functions being consider are continuous and standard functions. The effect the NSPPM has upon the N-world are standard effects produced by application of the standard part operator "st."

From the previous diagram, let F_1 and F_2 coincide and not separate. Call this location P and consider the diagram below. This is a three position classical NSPPM light-path and relative velocity diagram used for the infinitesimal light-clock analysis in section 6 of Article 2. This diagram is not a vector composition diagram but rather represents linear light-paths with respect to Einstein measures for relative velocities. It is also a relative velocity diagram to which hyperbolic "geometry" is applied.



Since Einstein measures are to be associated with this diagram, then this diagram should be obtained relative to infinitesimal light-clock counts and processes in the NSPPM. The three locations $F_1,\ F_2,\ P$ are assumed, at first, to coincide. When this occurs, the infinitesimal light-clock counts coincide. The object denoted by location P recedes from the $F_1,\ F_2$ locations with uniform NSPPM velocities, in standard form, of $\omega_1,\ \omega_2$, respectively. Further, consider the special case where both are observing the pulse sent from P at the exact some P-time. This produces the internal angle θ and exterior angle ϕ for this velocity triangle. The segments marked p_1 and p_2 are the projections of the velocity representations (not vectors) F_1P and F_2P onto the velocity representation F_1F_2. The n is the usual normal for this projection. We note that $p_1 + p_2 = \omega_3$. We apply hyperbolic trigonometry in accordance with [2], where we need to consider a particular k. We do this by scaling the velocities in terms of light units and let $k = c$. From [2, p. 143]

$$\begin{cases} \tanh(p_1/c) = (\tanh(\omega_1/c))\cos\theta \\ \tanh(p_2/c) = -(\tanh(\omega_2/c))\cos\phi \end{cases}, \tag{A4}$$

and also

$$\sinh(n/c) = (\sinh(\omega_1/c))\sin\theta = (\sinh(\omega_2/c))\sin\phi. \tag{A5}$$

Now, eliminating θ from $(A4)$ and $(A5)$ yields [1, p. 146]

$$\cosh(\omega_1/c) = (\cosh(p_1/c))\cosh(n/c). \tag{A6}$$

Combining (A4), (A5) and (A6) leads to the hyperbolic cosine law [2, p. 167].

$$\cosh(\omega_1/c) = (\cosh(\omega_2/c))\cosh(\omega_3/c) + (\sinh(\omega_2/c))(\sinh(\omega_3/c))\cos\phi. \tag{A7}$$

From $(A3)$, where each v_i is the Einstein relative velocity, we have that

$$e^{\omega_i/c} = \left(\frac{1 + v_i/c}{1 - v_i/c}\right)^{(1/2)}, i = 1, 2, 3. \tag{A3$'$}$$

From the basic hyperbolic definitions, we obtain from $(A3)'$

$$\begin{cases} \tanh(\omega_i/c) = v_i/c \\ \cosh(\omega_i/c) = (1 - v_i^2/c^2)^{-1/2} = \beta_i \\ \sinh(\omega_i/c) = \beta_i v_i/c \end{cases}. \tag{A8}$$

Our final hyperbolic requirement is to use

$$\tanh(\omega_3/c) = \tanh(p_1/c + p_2/c) = \frac{\tanh(p_1/c) + \tanh(p_2/c)}{1 + (\tanh(p_1/c))\tanh(p_2/c)}. \tag{A9}$$

Now into $(A9)$, substitute $(A4)$ and then substitute the first case from $(A8)$. One obtains

$$v_1\cos\theta = \frac{v_3 - v_2\cos\phi}{1 - \alpha},\ \alpha = \frac{v_3 v_2\cos\phi}{c^2}. \tag{A10}$$

Substituting into $(A7)$ the second and third cases from $(A8)$ yields

$$\beta_1 = \beta_2\beta_3(1 - \alpha),\ \beta_i = (1 - v_i^2/c^2)^{-1/2}. \tag{A11}$$

From equations $(A11)$, $(A5)$ and the last case in $(A8)$ is obtained

$$v_1\sin\theta = \frac{v_2\sin\phi}{\beta_3(1 - \alpha)}. \tag{A12}$$

For the specific physical behavior being displayed, the photons received from P at F_1 and F_2 are "reflected back" at the NSPPM P-time t^r. We then apply to this three point scenario our previous results. [Note: For comprehension, it may be necessary to apply certain relative velocity viewpoints such as from F_1 the point P is receding from F_1 and F_2 is receding from P. In this case, the NSPPM times when the photons are sent from F_1 and F_2 are related. Of course, as usual there is assumed to be no time delay between the receiving and the sending of a "reflected" photon.] In this case, let $t^{(1)},\ r^{(1)},\ v_1$ be the Einstein measures at F_1 for this P-event, and $t^{(2)},\ r^{(2)},\ v_2$ be the Einstein measures at F_2. Since $t^r = \beta_1^{-1}t^{(1)},\ t^r = \beta_2^{-1}t^{(2)}$ (p. 52), then

$$\frac{t^{(1)}}{\beta_1} = \frac{t^{(2)}}{\beta_2} \text{ and } r^{(1)} = v_1t^{(1)},\ r^{(2)} = v_2t^{(2)}. \tag{A13}$$

Suppose that we have the four coordinates, three rectangular, for this P event as measured from $F_1 = (x^{(1)}, y^{(1)}, z^{(1)}, t^{(1)})$ and from $F_2 = (x^{(2)}, y^{(2)}, z^{(2)}, t^{(2)})$ in a three point plane. It is important to recall that the x, y, z are related to Einstein measures of distance. Further, we take the x-axis as that of F_1F_2. The v_3 is the Einstein measure of the F_2 velocity as measured by an inf. light-clock at F_1. To correspond to the customary coordinate system employed [1, p. 32], this gives

$$\begin{cases} x^{(1)} = v_1t^{(1)}\cos\theta,\ y^{(1)} = v_1t^{(1)}\sin\theta,\ z^{(1)} = 0 \\ x^{(2)} = -v_2t^{(2)}\cos\phi,\ y^{(2)} = v_2t^{(2)}\sin\phi,\ z^{(2)} = 0 \end{cases}. \tag{A14}$$

It follows from $(A10), \cdots, (A14)$ that

$$t^{(1)} = \beta_3(t^{(2)} - v_3x^{(2)}/c^2),\ x^{(1)} = \beta_3(x^{(2)} - v_3t^{(2)}),\ y^{(1)} = y^{(2)},\ z^{(1)} = z^{(2)}. \tag{A15}$$

Hence, for this special case $\omega_1, \omega_2, \theta,\ \phi$ are eliminated and the Lorentz Transformations are established. If $P \neq F_1, P \neq F_2$, then the fact that $x^{(1)},\ x^{(2)}$ are not the measures for a physical ruler but are measures for a distance related to Einstein measures, which are defined by the properties of the propagation of electromagnetic radiation and infinitesimal light-clock counts, shows that the notion of actual natural world "length" contraction is false. For logical consistency, Einstein measures as determined by the light-clock counts are necessary. This analysis is relative to a "second" pulse when light-clock counts are considered. The positions F_1 and F_2 continue to coincide during the first pulse light-clock count determinations.

Infinitesimal light-clock counts allow us to consider a real interval as an interval for "time" measure as well as to apply infinitesimal analysis. This is significant when the line-element method in Article 3 is applied to determine alterations in physical behavior. All of the coordinates being considered must be as they would be understood from the Einstein measure viewpoint. The interpretations must always be considered from this viewpoint as well. Finally, the model theoretic error of generalization is eliminated by predicting alterations in clock behavior rather than by the error of inappropriate generalization.

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